Denoising of DORIS Stations Coordinates Time Series by Wavelet and the Singular Spectrum Analysis

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Abstract

The aim of this paper is to elaborate an adequate denoising methodology of stations coordinates time series of space geodesy in order to recuperate the original signal from the noised data, which allowed to better apprehend the temporal variability of the physical phenomena (deformations of the earth's crust, transfers mass, geodynamic phenomena local, etc.).

The denoising technique used in this study is based on two different approaches, applied to signals affected by a white noise. The wavelet transform in the frequency space, based on the thresholding of the wavelet coefficients, and the Singular Spectrum Analysis (SSA) in the phase space, based on the computation of the eigenvectors of the covariance matrix formed from the time series.

The adopted methodology was applied to the weekly time series of the East residual coordinates sets of DORIS (Doppler Orbitography and Radio-positioning Integrated by Satellite) stations, provided by IGN/JPL (Institut Géographique National/ Jet Propulsion Laboratory), referred to ITRF2000 and expressed in the local geodetic reference frame after removal of ITRF2000 model of positions and velocities. The obtained results show that the optimal method of denoising, based on the standard deviation minimization, is the SSA method. However, all studied time series are affected by a significant white noise explained by the diminution of their signal-to-noise ratio.

Denoising by Wavelet Transform

Assume that the observed data vector $X=[X_1,X_2,...X_N]^T$ is given by :

$$X_t = S_t + R_t \qquad t=1,2,...N$$

Where:

S_t is the true signal and R_t is Gaussian white noise with i.i.d distribution $N(0,\sigma)$.

$W(.)$ and $W^{-1}(.)$ denote the forward and inverse discrete wavelet transform operators.

$D(., \lambda)$ denote the denoising operator with threshold λ.

The denoising procedure proceeds in three steps :

1- Decompose : Compute the wavelet decomposition of the signal X at level l; $W=W(X)$.

2 – Threshold detail coefficient (wavelet coefficients); $Z=D(W,\lambda)$.

3 - Reconstruct : from the coefficients thresholding, one reconstruct the signal ; $S=W^{-1}(Z)$.

Threshold strategy (*Soft thresholding*)

$$T_\lambda^{Soft}(W)=\begin{cases} W-\lambda & \text{if } W \geq \lambda \\ W+\lambda & \text{if } W \leq -\lambda \\ 0 & \text{if } |W| \leq \lambda \end{cases}$$

Where: W are the wavelet coefficients and λ is threshold value (λ>0).

Determination of the threshold (*Universal threshold*)

$$\lambda=\hat{\sigma}\sqrt{2\log(N)} \qquad \hat{\sigma}=\frac{Med}{0.6745}$$

Where:

N is the number of measurements and σ^2 is the noise variance.

Med is the median of the absolute values of the detail coefficients at the finest scale.

0.6745 is selected following a calibration with Gaussian distribution.

Denoising by SSA

The SSA method allows to extract significant components from time series (trends, periodical component and noise). The method is based on the computation of the eigenvalues and the eigenvectors of a covariance matrix C formed from the time series {Xi, i=1..., N} and the reconstruction of this time series based on a number of selected eigenvectors associated with the significant eigenvalues of the covariance matrix C.

The algorithm of SSA includes the following four steps:

Step (1): *Choice of the embedding dimension M*

Step (2): *Computation of the M x M covariance matrix C given by*:

$$C=\frac{1}{N'}D^tD \qquad D=\begin{pmatrix} X_1 & X_2 & \cdots & X_M \\ X_2 & X_3 & \cdots & X_{M+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N-M+1} & X_{N-M+2} & \cdots & X_N \end{pmatrix}$$

Where: N is the length of the time series and N' = N – M + 1.

Step (3): *Study of the eigenvalues of the covariance matrix C*

If we arrange and plot the ordered eigenvalues by decreasing value, one can often conclude that:
- The signal has a trend if the diagram contains an isolated eigenvalues,
- The signal is periodic if there are two close eigenvalues that have the same dominant frequency,
- The small eigenvalues constitute the noise of the signal.

Step (4): *Projection of the original time series onto the k-th eigenvectors $\{E_k, 1\leq k\leq M\}$ and its reconstruction*

$$R_\kappa(t)=\begin{cases} \frac{1}{t}\sum_{k\in\kappa}\sum_{j=1}^{t}A_k(t-j+1)E_{k,j}, & 1\leq t\leq M-1 \\ \frac{1}{M}\sum_{k\in\kappa}\sum_{j=1}^{M}A_k(t-j+1)E_{k,j}, & M\leq t\leq N' \\ \frac{1}{N-t+1}\sum_{k\in\kappa}\sum_{j=t-N+M}^{M}A_k(t-j+1)E_{k,j}, & N'+1\leq t\leq N \end{cases} \qquad A_k(t)=\sum_{k=1}^{M}X(t+j-1)E_{k,j}$$

Data description

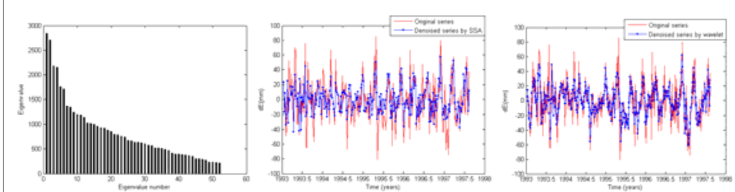
- Weekly solutions of residual coordinates sets of DORIS stations,
- provided by IGN/JPL (Institut Géographique National, France/ Jet Propulsion Laboratory, USA),
- referred to ITRF2000, and
- expressed in the local geodetic reference frame (dN: North component, dE: East component and dU: Up component), after removal of the ITRF2000 model of positions and velocities.

Acronym	Name	Country	Lat	Lon	Data span	Observations number
TLSA	Toulouse	France	43.56	1.48	1993-1997.6	239
LIBB	Libreville	Gabon	0.35	9.67	1999.2-2003.6	231
SYPB	Syowa	l'Antarctique	-69°	39.58°	1999.2-2003.6	225

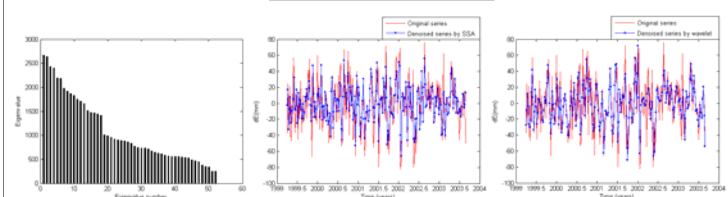
Table 1: DORIS stations selected in this study

Results

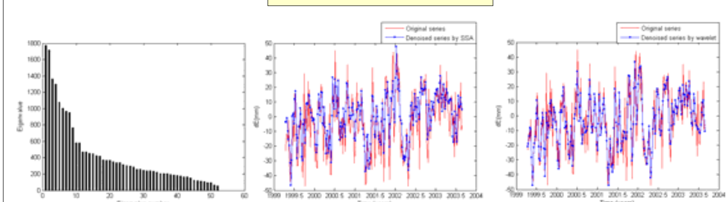
Station TLSA



Station LIBB



Station SYPB



Station	Time series	Minimum	Maximum	Mean	STD	SNR
TLSA	Original series	-81	85	-1	30	
	Denoised series by wavelet	-61	62	-1	22	3.32
	Denoised series by SSA	-44	57	0	19	3.44
LIBB	Original series	-82	76	-1	32	
	Denoised series by wavelet	-71	72	-1	24	3.66
	Denoised series by SSA	-66	56	0	22	4.46
SYPB	Original series	-47	45	-4	20	
	Denoised series by wavelet	-47	37	-4	17	5.47
	Denoised series by SSA	-46	47	0	16	4.88

Table 2: Statistics of denoised time series by wavelet and SSA [mm]

Conclusion

The application of wavelet and SSA method to the weekly time series of the East residual coordinates of three DORIS station (TLSA, LIBB and SYPB) permits to better reduce their noise. The obtained results show that the optimal method of denoising, based on the minimization of the standard deviation, is the SSA method. Indeed, The standard deviation (STD) of the denoised time series of the three stations (TLSA, LIBB and SYPB) are about 19, 22 and 16mm, respectively, compared to the 30, 32 and 20mm of the original ones, consequently the reduction of the noise is thus rather considerable. However, the signal-to-noise ratio (SNR), computed in decibels (dB), of the time series (dE) of station SYPB is about 4.88dB, compared to 3.44dB of TLSA time series and 4.46dB of LIBB time series. What shows that the East component of the station SYPB is the least disturbed than the two others, which due to the near polar orbit of the SPOT and ENVISAT satellites (98° inclination). For sites at high latitude like SYPB, the satellite passes are less orthogonal to this direction and therefore, the high latitude stations are globally more observed than the others by SPOT and ENVISAT satellites and less observed by TOPEX/Poseidon (66.04° inclination).

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