Combination at the Observation Level
IERS Working Group

Multi-technique combination for Earth Orientation & Reference Frames Determination

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and

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Combination at the Observation Level

Why combining at the Observation Level

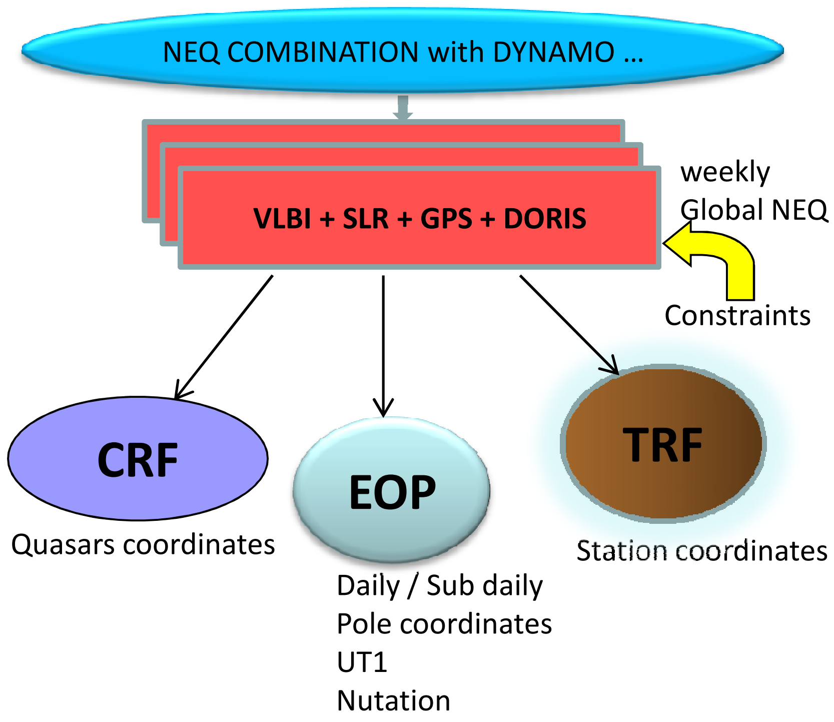
- Space geodetic techniques have different strengths and weaknesses for recovering geodetic parameters
- Some systematic behaviour which can easier and more efficiently be detected and reduced at the observation level.

Goal

- This could contribute to the IERS scope for a rigorous combination of ITRF, EOP and ICRF and ZTD.

How

- At the same epoch the observation equations from 4 space geodetic techniques GNSS, VLBI, SLR and DORIS are weekly stacked.
- Combination processes can be performed to determine common geodetic parameters



Combination at the Observation level or at the Normal Equation level

Actual equivalence between observation level and NEq level:

➔Only technique independent parameters can be reduced

Test: Jason2 - 7 day arc

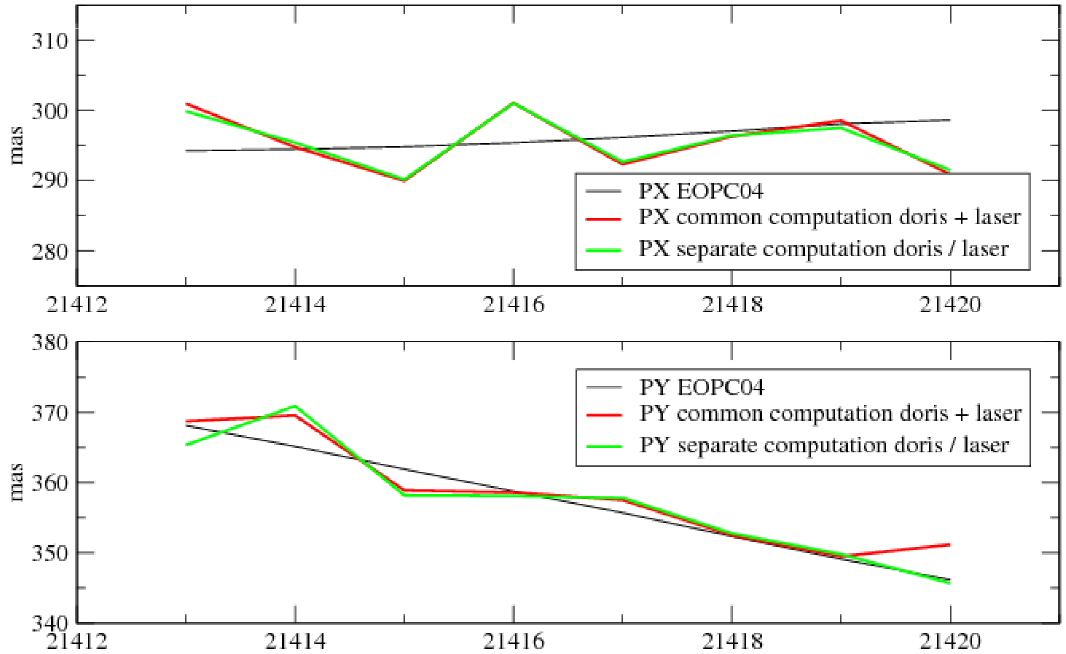
over the period

17/8/2008 – 23/8/2008

Technique	Nb of obs / eliminated obs	Residuals	Orbit # rms
SLR NEq	2216 / 224	4.1cm	12.1cm
DORIS NEq	109884 / 52825	.346mm/s	10.6cm
SLR + DORIS NEq	2247 / 193 109614 / 53095	4.2cm .352mm/s	

X-Pole

Y-Pole



Unconstrained Normal Equation

$$\underbrace{A^T\Pi A}_{N}.\Delta p = \underbrace{A^T\Pi.\Delta Q}_{S} \rightarrow N.\Delta p = S$$

$$\Pi = \begin{pmatrix} \frac{1}{\sigma_1^2} & 0 & . & . & 0 \\ 0 & \frac{1}{\sigma_2^2} & . & . & 0 \\ . & . & . & . & . \\ 0 & . & . & \frac{1}{\sigma_{n-1}^2} & 0 \\ 0 & 0 & . & 0 & \frac{1}{\sigma_n^2} \end{pmatrix}$$

weigth Matrix with σ_i^2 uncertainty of observation i

NEq Reduction	NEq Weighted & Cumulated
$\begin{pmatrix} N_{II} & N_{IE}^T \\ N_{IE} & N_{EE} \end{pmatrix}\begin{pmatrix} \Delta p_I \\ \Delta p_E \end{pmatrix} = \begin{pmatrix} S_I \\ S_E \end{pmatrix}$	for the i[th] NEq: $N_i = A_i^T\Pi_i A_i$ and RHS term $S_i = A_i^T\Pi_i\Delta Q_i$
$\begin{cases} N_{II}\Delta p_I + N_{IE}^T\Delta p_E = S_I \\ N_{IE}\Delta p_I + N_{EE}\Delta p_E = S_E \end{cases}$	$N_i.\Delta p = S_i$
$\Delta p_I = N_{II}^{-1}\left(S_I - N_{IE}^T\Delta p_E\right)$	π_i the weighting associated to N_i
$N_{IE}N_{II}^{-1}\left(S_I - N_{IE}^T\Delta p_E\right) + N_{EE}\Delta p_E = S_E$	$\underbrace{\left(\sum_{i=1}^{k}\pi_i N_i\right)}_{N}\Delta p = \underbrace{\left(\sum_{i=1}^{k}\pi_i S_i\right)}_{S}$
$\underbrace{\left(N_{EE} - N_{IE}N_{II}^{-1}N_{IE}^T\right)}_{N^{\otimes}}\Delta p_E = \underbrace{\left(S_E - N_{IE}N_{II}^{-1}S_I\right)}_{S^{\otimes}}$	$N.\Delta p = S$
$N^{\otimes}.\Delta p_E = S^{\otimes}$ the reduced matrix	$\pi_i = \frac{n_i}{\Delta Q_i^T\Pi_i\Delta Q_i}$, with n_i = nb obs. of i set
	Π_i = diagonal matrix $1/\sigma_j^2$ for $j = 1..n_i$
	or
	π_i = calculated by iterative Helmert algorithm

Constrained Normal Equation

Continuity constraints on EOP

$$\Delta eop(t) - \Delta eop(t - \Delta t) = 0 \pm \sigma_{\text{constraint}}$$

Stability constraints on station coordinates (X_S,Y_S,Z_S)

$$\alpha_S \Delta X_S + \beta_S \Delta Y_S + \gamma_S \Delta Z_S = 0 \pm \sigma_S$$

Minimal constraints on transformation parameters

Translation
Scale
Rotation

$$\begin{cases} T_X = 0 + \sigma_{\min}, T_Y = 0 + \sigma_{\min}, T_Z = 0 + \sigma_{\min} \\ D = 0 + \sigma_{\min} \\ R_X = 0 + \sigma_{\min}, R_Y = 0 + \sigma_{\min}, R_Z = 0 + \sigma_{\min} \end{cases}$$

Local ties constraints on stations coordinates *i* and *j*

$$\begin{cases} \Delta X_{Si} - \Delta X_{Sj} = \Delta X_{Si-Sj} + \sigma_{Si-Sj} \\ \Delta Y_{Si} - \Delta Y_{Sj} = \Delta Y_{Si-Sj} + \sigma_{Si-Sj} \\ \Delta Z_{Si} - \Delta Z_{Sj} = \Delta Z_{Si-Sj} + \sigma_{Si-Sj} \end{cases}$$

These constraints are pseudo observations converted in normal equation

$$N_c . \Delta p = S_c$$

and added to the normal equation of observations

$$\left(N + N_c\right) . \Delta p = S + S_c$$

Normal Equation Resolution

Determination of geodetic parameters using the Normal Equation Level

$$\left(\Delta\hat{p}\right) = N^{-1}S$$

Inverse Methods:

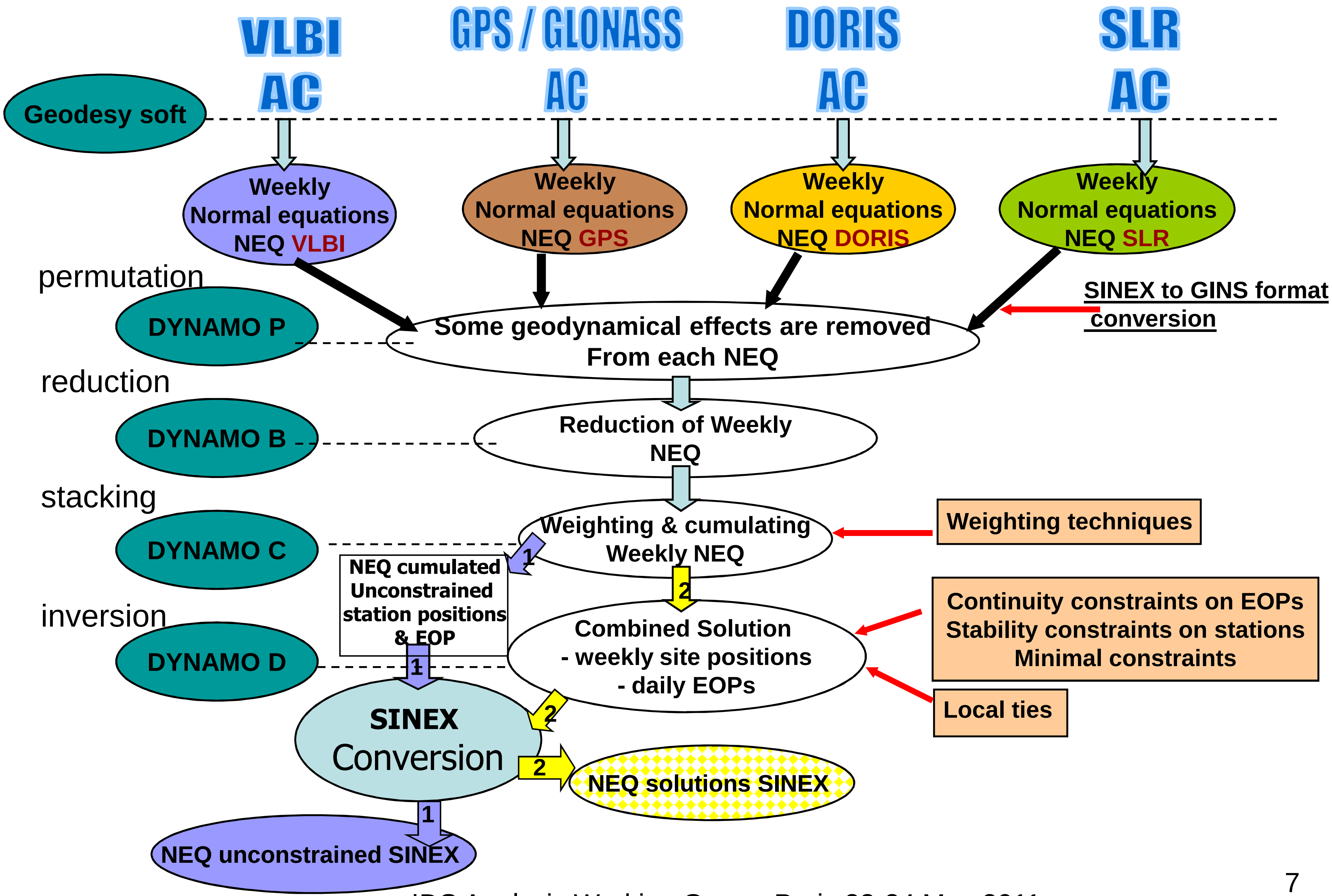
Explicit resolution

- Choleski
- Conjugate Gradients
- Singular Value Decomposition
- QR decomposition
  - Householder orthogonal transformations
  - Givens rotation matrix

Iterative resolution

- Jacobi iteration
- Gauss-Seidel iteration
- Relaxation method

GRGS NEq Combination processing



Working Group on Combination at the Observation Level (**COL**)

Created in the frame of the IERS in October 2009

WG-COL Objectives

- Study methods and advantages of combining techniques (DORIS, GPS, SLR, VLBI) at the observation level
- Improve resolution and consistency of products (EOP, TRF, CRF) to increase accuracy of parameter determination
- Study technique dependent systematic errors
- Progress in combination methods and strategies (eg. weighting)
- Creating common standards for a rigorous combination
- Mutualize physical parameters (eg. troposphere)
- Extend the combination approach at the level of observation to several research groups in a planned IERS action
- Validate the rigorous combination approach vs. present realizations (C04, ITRF…)
- Prepare future of IERS

Working Group on Combination at the Observation Level (**COL**)

Project

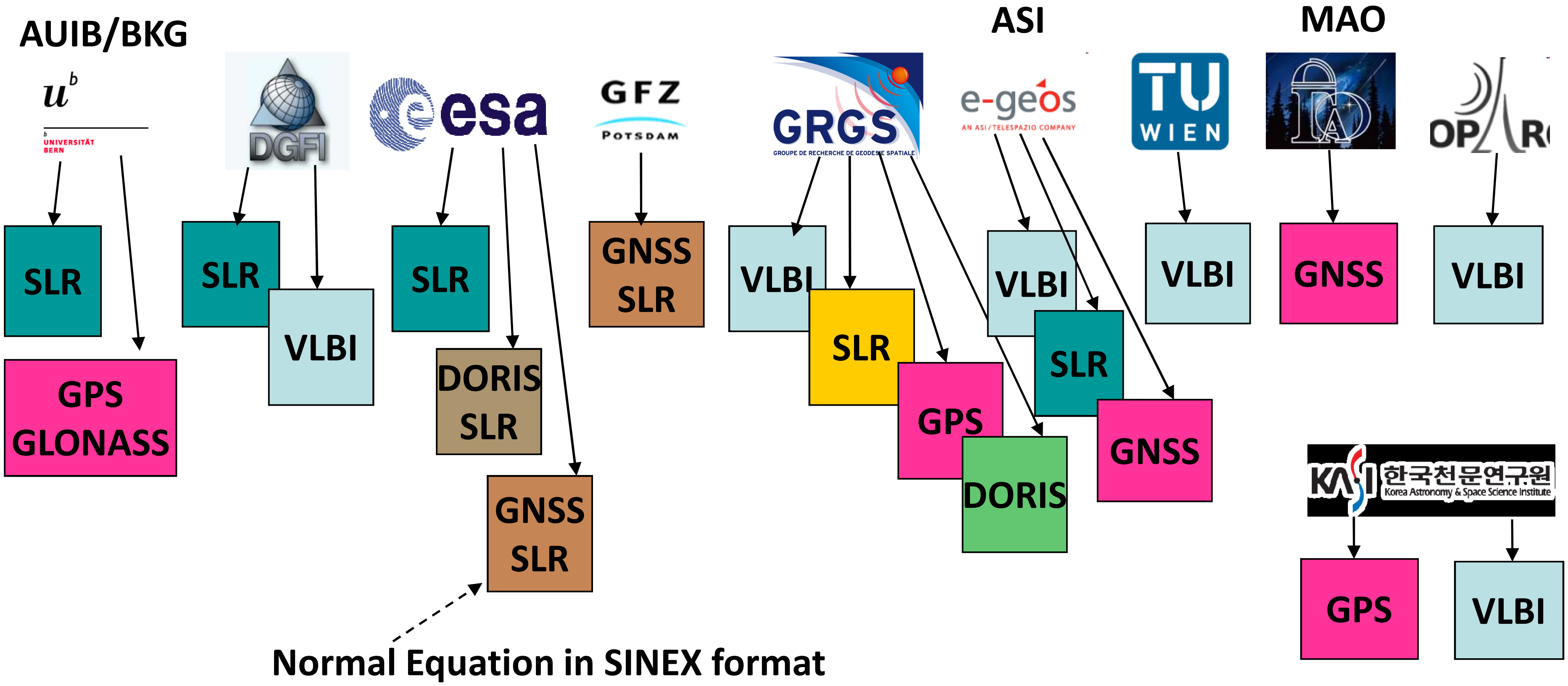
- Compare the EOP solutions per technique & combined multi-techniques
- Compare the Stations Coordinates solutions
- Compare heterogeneous Software by inter-comparing results in parameters estimations

Benchmark

- August 10 to August 30, 2008 including the CONT08 VLBI period (12-26/08/08)
- based on weekly combined SINEX files from all space geodetic techniques together containing normal equations
- parameters

  Geodetic Station coordinates
  Polar motion
  Nutation parameters
  UT1
  eventually Quasar coordinates and troposphere parameters

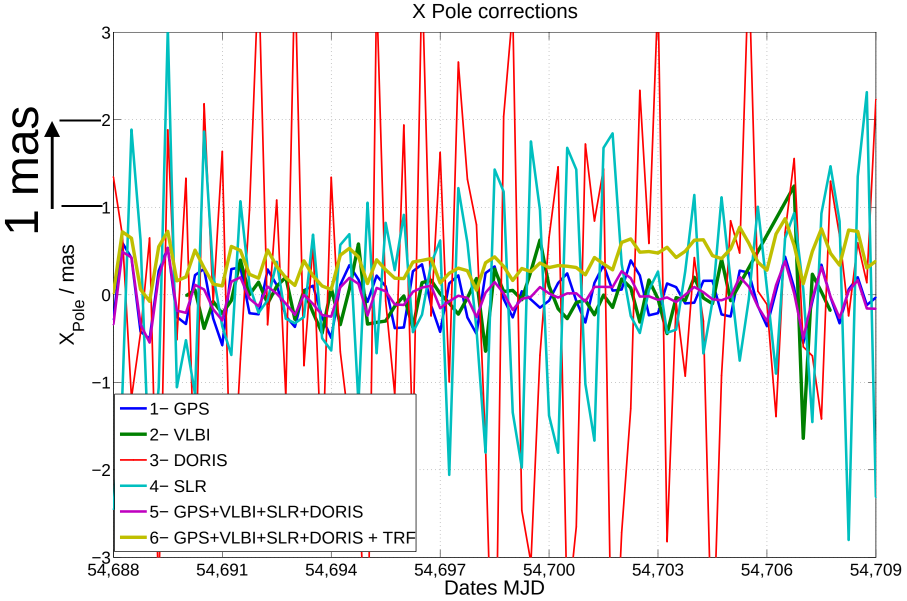
Centers participating to the COL comparison campaign and techniques performed per center

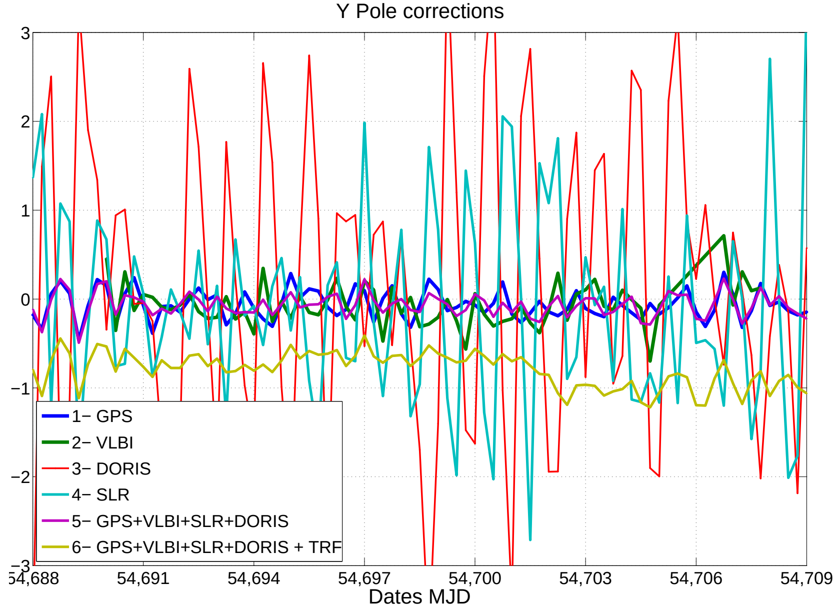


COL participants & softwares

Analysis Centers	Software
AIUB	***BERNESE*** software as used by IGS AC CODE for GNSS
BKG	***GEODYN*** software for SLR ***BERNESE*** + GIPSY software for GNSS ***CALC*** / ***SOLVE*** for VLBI
DGFI	***DOGS*** 5.0 software for SLR, ***OCCAM*** 6.1 LSM + ***DOGS*** 5.0 for VLBI
ESOC	***NAPEOS*** software
GFZ	***EPOSOC*** 06.61 software
GRGS	***GINS*** / ***DYNAMO*** software
ASI	***GEODYN*** software
TUW	***VieVS*** software for VLBI
MAO	***CoCos*** Construct Combined Solution Software
JPL (potentially)	***GIPSY*** / ***OASIS***
GSFC (potentially)	***GEODYN*** / ***SOLVE***
Korea Astro Space Science Institute	

X and Y pole corrections from GRGS NEqs versus C04 series at 6h intervals over 3 weeks



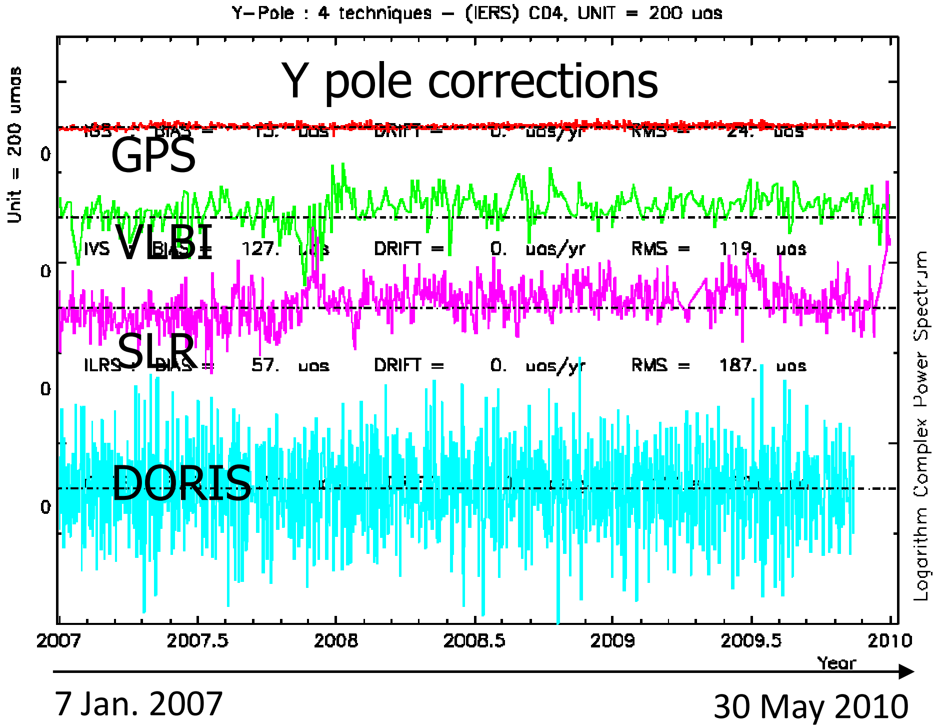


X_ Pole / µas	Technique	Weighted Mean	Weighted RMS
	1- GPS	-10.3	160.6
	2- VLBI	-17.2	188.9
	3- DORIS	31.5	1484.7
	4- SLR	-25.7	855.2
	5- Combined	-10.0	165.6
	6- Combined +TRF	379.8	185.0

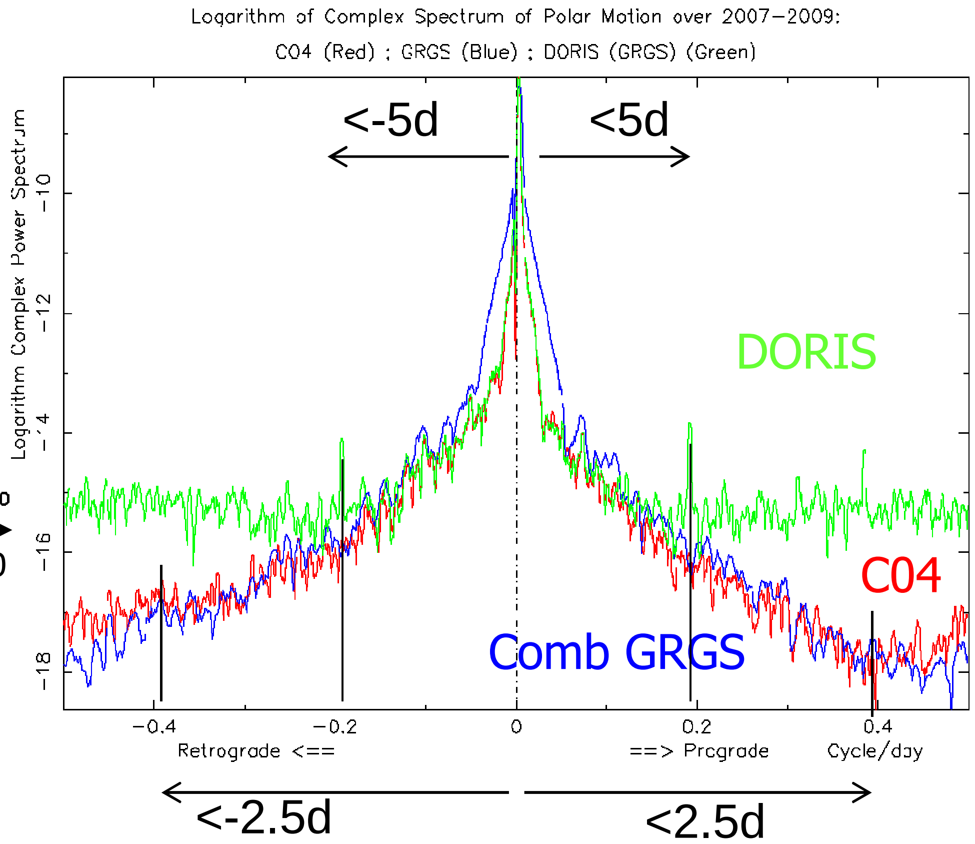
Y_ Pole / µas	Technique	Weighted Mean	Weighted RMS
	1- GPS	-60.3	116.8
	2- VLBI	-91.7	174.2
	3- DORIS	262	1098.2
	4- SLR	-193	799.7
	5- Combined	-66.8	106.2
	6- Combined +TRF	-794	211.48

Daily Pole coordinates from DORIS technique versus C04 series at 1d intervals

X & Y pole series 1050 days -7 January 2007 to 30 May 2010



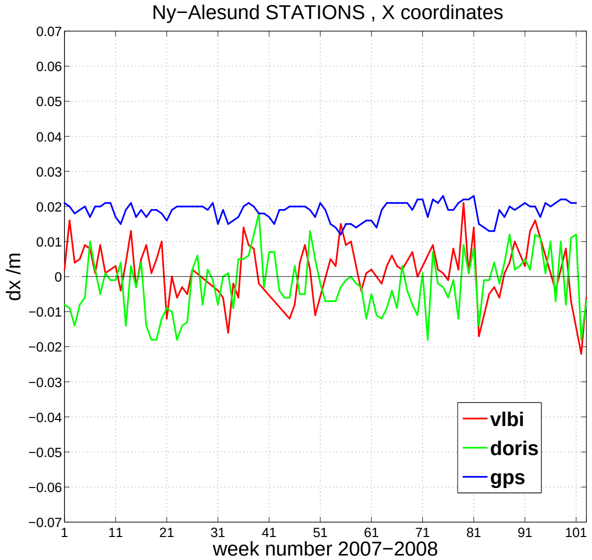
	DORIS
X pole mean	96 µas
Y pole mean	74 µas
X pole WRMS	650 µas
Y pole WRMS	519 µas

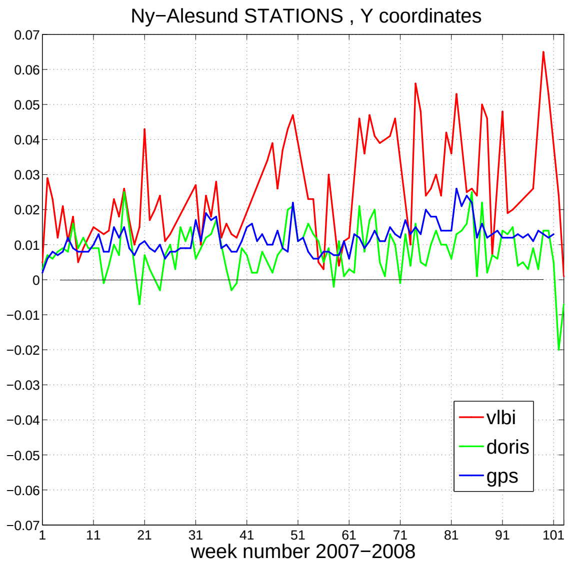


Station Coordinates
VLBI DORIS GPS Combination

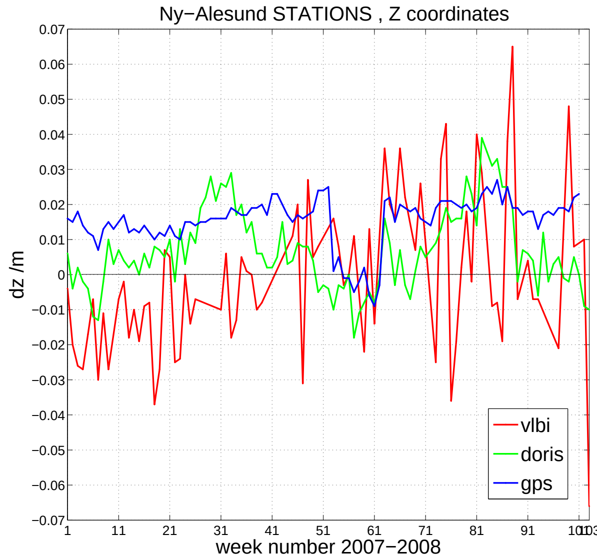
Solutions by inversion of weekly unconstrained combined GPS SLR DORIS VLBI NEQ
Pole UT & Nutation are fixed to the a priori **EOP C04 series**
Weekly Terrestrial Station Coordinates are obtained with
Systematic Constraints : σ(Sx,Sy,Sz) = 1m
Minimal Constraints
Ties Constraints on 26 co-located sites

Site Ny-Alesund : 3 space geodetic techniques co-located, VLBI, GPS, DORIS





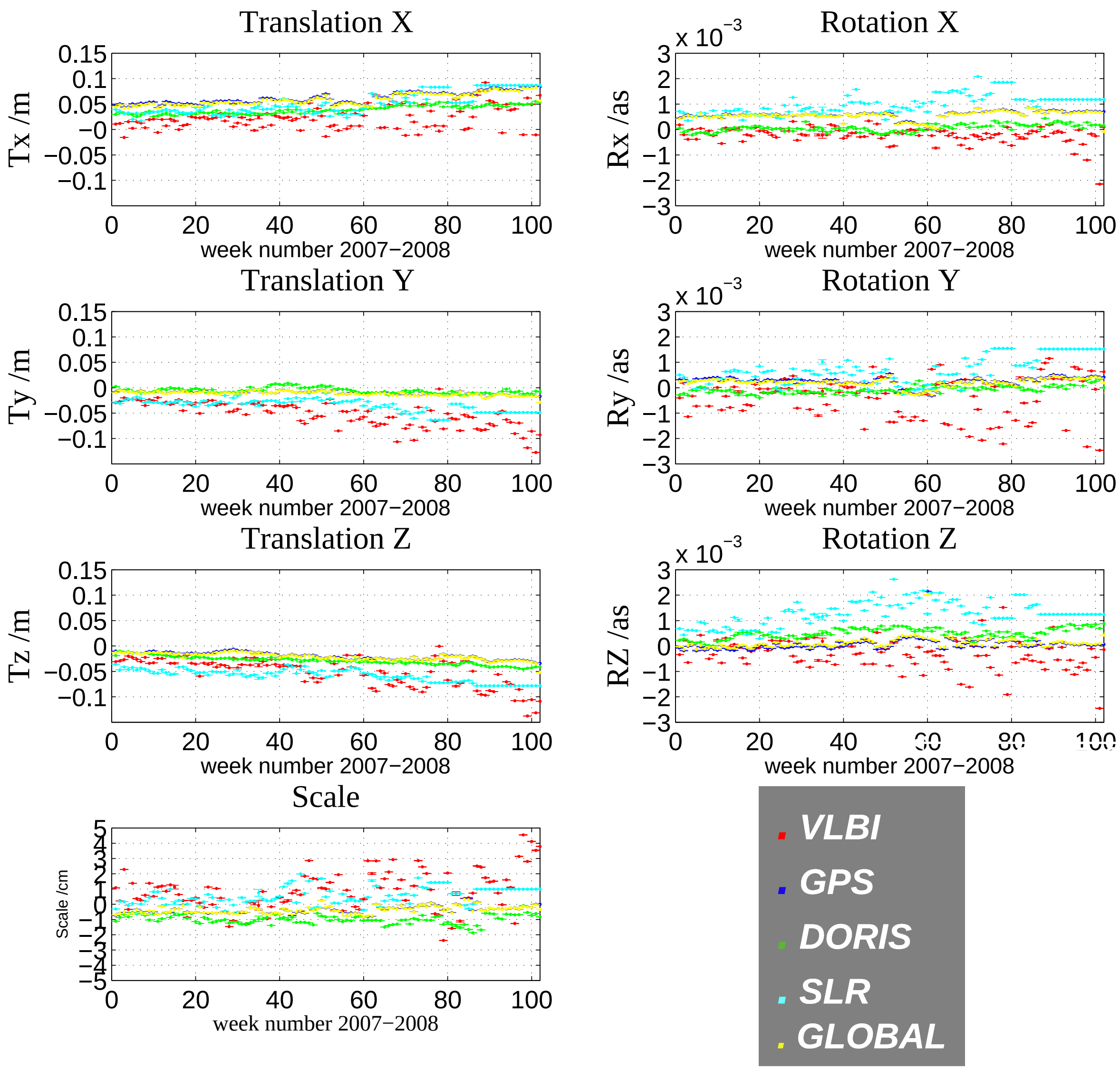
2007.0 - 2009.0



	$\bar{X}$ / mm	$\bar{Y}$ / mm	$\bar{Z}$ / mm	$\langle (X-\bar{X})^2 \rangle$ / mm	$\langle (Y-\bar{Y})^2 \rangle$ / mm	$\langle (Z-\bar{Z})^2 \rangle$ / mm	$d\bar{X}$ / mm/y	$d\bar{Y}$ / mm/y	$d\bar{Z}$ / mm/y
VLBI	+ 2.0	+25.3	- 1.1	7.9	14.7	22.2	- 2.3	+19.5	+52.5
GPS	+18.8	+11.8	+15.4	2.5	4.2	6.9	- 4.0	+ 5.1	- 4.9
DORIS	- 2.2	+ 8.2	+ 6.9	8.1	7.0	11.4	+ 1.5	+ 6.7	+13.2

Transformation Parameters for GPS DORIS VLBI SLR and for GLOBAL combination with respect to ITRF2008

S



Transformation Parameters

Mean	DORIS Mean / σ	COMB Mean / σ
Tx cm	**+3.9 / 0.8**	**+5.9 / 1.1**
Ty cm	**-0.6 / 0.5**	**-1.1 / 0.4**
Tz cm	**-2.9 / 0.8**	**-2.1 / 0.7**
D cm	**-0.99 / 0.29**	**-0.38 / 0.24**
Rx µas	**+58.8 / 144**	**545 / 176**
Ry µas	**-75.3 / 153**	**199 / 171**
Rz µas	**+507 / 208**	**119 / 237**

Rate	DORIS Rate / σ	COMB Rate / σ
Tx cm/y	**+1.3 / 0.1**	**+1.7 / 0.2**
Ty cm/y	**-0.4 / 0.1**	**-0.5 / 0.1**
Tz cm/y	**-1.3 / 0.8**	**-1.1 / 0.1**
D cm/y	**-0.05/ 0.10**	**0.20 / 0.07**
Rx µas/y	**156 / 39**	**62 / 59**
Ry µas/y	**169 / 40**	**17 / 59**
Rz µas/y	**213 / 58**	**126 / 78**

COL-WG Prospect

- **Re-iterate the CONT08 campaign** with homogenized standards and parameters,
  a priori reference system ITRF2008, ICRF2, EOP-C04, IERS conventions 2010
- **NEq multi-techniques combination** weekly bases on CONT08 campaign for several
  participating ACs have to be compared
  multi-technique combination from different ACs have to be performed
- **New sets of data**: LEO satellites
  Jason-2 (SLR,DORIS,GPS multi-techniques on board)
  GRACE (SLR,GPS two techniques on board)
- **Study the sub-diurnal EOP** variations by deriving hourly or 2-hours estimates
- **Interpolation method** of a-priori EOPC04 for the data epoch has to be adopted
- The next **participation meetings**
  -25th general assembly of the IUGG (28 June - 07 July 2011)
  -3rd international colloquium “Scientific and Fundamental Aspects of the Galileo Program” (31 August 02 September 2011)
  -“Journées de référence spatio-temporels” Paris September 2011
  -3rd COL-WG meeting will be held at Paris (October 2011)
- **GRASP** space mission: “**G**eodetic **R**eference **A**ntenna in **S**pace” NASA’s Jet Propulsion Laboratory project will carry precise sensors system for GNSS, SLR, DORIS and VLBI geodetic techniques on space board satellite. The launch is expected in 2017.
- **HY-2A** satellite mission using the SLR GPS and DORIS technique on board

Thanks for your attention