Multi-technique combination for Earth Orientation & Reference Frames Determination

Jean-Yves Richard(1)
and
Daniel Gambis(1), Christian Bizouard(1), Richard Biancale(2), Géraldine Bourda(3), Florent Deleflie(4), Sylvain Loyer(5), Laurent Soudarin(5)

(1) Observatoire de Paris
(2) CNES, Toulouse
(3) Observatoire de Bordeaux
(4) IMCCE, Lille
(5) CNES-CLS, Toulouse

Combination at the Observation Level

Why combining at the Observation Level
• Space geodetic techniques have different strengths and weaknesses for recovering geodetic parameters
• Some systematic behaviour which can easier and more efficiently be detected and reduced at the observation level.

Goal
• This could contribute to the IERS scope for a rigorous combination of ITRF, EOP and ICRF and ZTD.

How
• At the same epoch the observation equations from 4 space geodetic techniques GNSS, VLBI, SLR and DORIS are weekly stacked.
• Combination processes can be performed to determine common geodetic parameters
Combination at the Observation level or at the Normal Equation level

Actual equivalence between observation level and NEq level:

➔ Only technique independent parameters can be reduced

Test: Jason2 - 7 day arc
over the period

<table>
<thead>
<tr>
<th>Technique</th>
<th>Nb of obs / eliminated obs</th>
<th>Residuals</th>
<th>Orbit # rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLR NEq</td>
<td>2216 / 224</td>
<td>4.1cm</td>
<td>12.1cm</td>
</tr>
<tr>
<td>DORIS NEq</td>
<td>109884 / 52825</td>
<td>.346mm/s</td>
<td>10.6cm</td>
</tr>
<tr>
<td>SLR + DORIS NEq</td>
<td>2247 / 193</td>
<td>4.2cm</td>
<td>.352mm/s</td>
</tr>
<tr>
<td>SLR + DORIS NEq</td>
<td>109614 / 53095</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

X-Pole

Y-Pole

### Unconstrained Normal Equation

\[
A^T \Pi A \Delta p = A^T \Pi \Delta Q \rightarrow N \Delta p = S
\]

<table>
<thead>
<tr>
<th>NEq Reduction</th>
<th>NEq Weighted &amp; Cumulated</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
N_{II} & N_{IE}^T \\
N_{IE} & N_{EE}
\end{pmatrix}
\begin{pmatrix}
\Delta p_I \\
\Delta p_E
\end{pmatrix}
= \begin{pmatrix} S_I \\ S_E \end{pmatrix}
\]
| for the \(i^{th}\) NEq: \(N_i = A^T_i \Pi_i A_i\) and RHS term \(S_i = A^T_i \Pi_i \Delta Q_i\) |
| \(N_i \Delta p = S_i\) | \(N_i \Delta p = S\) |
| \(\pi_i\) the weighting associated to \(N_i\) | \(\pi_i = \frac{n_i}{\Delta Q_i^T \Pi_i \Delta Q_i}\), with \(n_i = \) nb obs. of \(i\) set |
| \[
\begin{pmatrix}
N_{II} \Delta p_I + N_{IE}^T \Delta p_E = S_I \\
N_{IE} \Delta p_I + N_{EE} \Delta p_E = S_E
\end{pmatrix}
\] | \(\Pi_i = \) diagonal matrix \(1/\sigma_j^2\) for \(j = 1..n_i\) |
| \[
\Delta p_I = N_{II}^{-1} \left( S_I - N_{IE}^T \Delta p_E \right)
\] | \(\pi_i = \) calculated by iterative Helmert algorithm |
| \[
N_{IE} N_{II}^{-1} \left( S_I - N_{IE}^T \Delta p_E \right) + N_{EE} \Delta p_E = S_E
\] | |
Constrained Normal Equation

### Continuity constraints on EOP
\[
\Delta eop(t) - \Delta eop(t - \Delta t) = 0 \pm \sigma_{\text{constraint}}
\]

### Stability constraints on station coordinates \((X_s, Y_s, Z_s)\)
\[
\alpha_s \Delta X_s + \beta_s \Delta Y_s + \gamma_s \Delta Z_s = 0 \pm \sigma_s
\]

### Minimal constraints on transformation parameters
- **Translation**
  \[
  T_X = 0 + \sigma_{\text{min}}, \quad T_Y = 0 + \sigma_{\text{min}}, \quad T_Z = 0 + \sigma_{\text{min}}
  \]
- **Scale**
  \[
  D = 0 + \sigma_{\text{min}}
  \]
- **Rotation**
  \[
  R_X = 0 + \sigma_{\text{min}}, \quad R_Y = 0 + \sigma_{\text{min}}, \quad R_Z = 0 + \sigma_{\text{min}}
  \]

### Local ties constraints on stations coordinates \(i\) and \(j\)
\[
\begin{align*}
\Delta X_i - \Delta X_j &= \Delta X_{i-j} + \sigma_{i-j} \\
\Delta Y_i - \Delta Y_j &= \Delta Y_{i-j} + \sigma_{i-j} \\
\Delta Z_i - \Delta Z_j &= \Delta Z_{i-j} + \sigma_{i-j}
\end{align*}
\]

\[
(N + N_c) \Delta p = S + S_c
\]

These constraints are pseudo observations converted in normal equation and added to the normal equation of observations.
Normal Equation Resolution

Determination of geodetic parameters using the Normal Equation Level

\[
(\Delta \hat{p}) = N^{-1} S
\]

**Inverse Methods:**

*Explicit resolution*
- Choleski
- Conjugate Gradients
- Singular Value Decomposition
- QR decomposition
  - Householder orthogonal transformations
  - Givens rotation matrix

*Iterative resolution*
- Jacobi iteration
- Gauss-Seidel iteration
- Relaxation method
Some geodynamical effects are removed from each NEQ. Reduction of Weekly NEQ.

Weighting & cumulating Weekly NEQ.

Combined Solution:
- weekly site positions
- daily EOPs

Local ties

Continuity constraints on EOPs
Stability constraints on stations
Minimal constraints

NEQ solutions SINEX

Working Group on Combination at the Observation Level (COL)

Created in the frame of the IERS in October 2009

WG-COL Objectives

- Study methods and advantages of combining techniques (DORIS, GPS, SLR, VLBI) at the observation level
- Improve resolution and consistency of products (EOP, TRF, CRF) to increase accuracy of parameter determination
- Study technique dependent systematic errors
- Progress in combination methods and strategies (eg. weighting)
- Creating common standards for a rigorous combination
- Mutualize physical parameters (eg. troposphere)
- Extend the combination approach at the level of observation to several research groups in a planned IERS action
- Validate the rigorous combination approach vs. present realizations (C04, ITRF...)
- Prepare future of IERS
Working Group on Combination at the Observation Level (COL)

Project

• Compare the EOP solutions per technique & combined multi-techniques
• Compare the Stations Coordinates solutions
• Compare heterogeneous Software by inter-comparing results in parameters estimations

Benchmark

• August 10 to August 30, 2008 including the CONT08 VLBI period (12-26/08/08)
• based on weekly combined SINEX files from all space geodetic techniques together containing normal equations
• parameters
  Geodetic Station coordinates
  Polar motion
  Nutation parameters
  UT1
  eventually Quasar coordinates and troposphere parameters
Centers participating to the COL comparison campaign and techniques performed per center

Normal Equation in SINEX format

## COL participants & softwares

<table>
<thead>
<tr>
<th>Analysis Centers</th>
<th>Software</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AIUB</strong></td>
<td><strong>BERNESE</strong> software as used by IGS AC CODE for GNSS</td>
</tr>
<tr>
<td><strong>BKG</strong></td>
<td><strong>GEODYN</strong> software for SLR</td>
</tr>
<tr>
<td></td>
<td><strong>BERNESE</strong> + GIPSY software for GNSS</td>
</tr>
<tr>
<td></td>
<td><strong>CALC / SOLVE</strong> for VLBI</td>
</tr>
<tr>
<td><strong>DGFI</strong></td>
<td><strong>DOGS</strong> 5.0 software for SLR,</td>
</tr>
<tr>
<td></td>
<td><strong>OCCAM</strong> 6.1 LSM + <strong>DOGS</strong> 5.0 for VLBI</td>
</tr>
<tr>
<td><strong>ESOC</strong></td>
<td><strong>NAPEOS</strong> software</td>
</tr>
<tr>
<td><strong>GFZ</strong></td>
<td><strong>EPOSOC</strong> 06.61 software</td>
</tr>
<tr>
<td><strong>GRGS</strong></td>
<td><strong>GINS / DYNAMO</strong> software</td>
</tr>
<tr>
<td><strong>ASI</strong></td>
<td><strong>GEODYN</strong> software</td>
</tr>
<tr>
<td><strong>TUW</strong></td>
<td><strong>VieVS</strong> software for VLBI</td>
</tr>
<tr>
<td><strong>MAO</strong></td>
<td><strong>CoCos</strong> Construct Combined Solution Software</td>
</tr>
<tr>
<td><strong>JPL (potentially)</strong></td>
<td><strong>GIPSY / OASIS</strong></td>
</tr>
<tr>
<td><strong>GSFC (potentially)</strong></td>
<td><strong>GEODYN / SOLVE</strong></td>
</tr>
<tr>
<td><strong>Korea Astro Space Science Institute</strong></td>
<td></td>
</tr>
</tbody>
</table>
X and Y pole corrections from GRGS NEqs versus C04 series at 6h intervals over 3 weeks

<table>
<thead>
<tr>
<th>Technique</th>
<th>Weighted Mean</th>
<th>Weighted RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X Pole</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1- GPS</td>
<td>-10.3</td>
<td>160.6</td>
</tr>
<tr>
<td>2- VLBI</td>
<td>-17.2</td>
<td>188.9</td>
</tr>
<tr>
<td>3- DORIS</td>
<td>31.5</td>
<td>1484.7</td>
</tr>
<tr>
<td>4- SLR</td>
<td>-25.7</td>
<td>855.2</td>
</tr>
<tr>
<td>5- Combined</td>
<td>-10.0</td>
<td>165.6</td>
</tr>
<tr>
<td>6- Combined +TRF</td>
<td>379.8</td>
<td>185.0</td>
</tr>
<tr>
<td><strong>Y Pole</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1- GPS</td>
<td>-60.3</td>
<td>116.8</td>
</tr>
<tr>
<td>2- VLBI</td>
<td>-91.7</td>
<td>174.2</td>
</tr>
<tr>
<td>3- DORIS</td>
<td>262</td>
<td>1098.2</td>
</tr>
<tr>
<td>4- SLR</td>
<td>-193</td>
<td>799.7</td>
</tr>
<tr>
<td>5- Combined</td>
<td>-66.8</td>
<td>106.2</td>
</tr>
<tr>
<td>6- Combined +TRF</td>
<td>-794</td>
<td>211.48</td>
</tr>
</tbody>
</table>
Daily Pole coordinates from DORIS technique versus C04 series at 1d intervals

X & Y pole series 1050 days -7 January 2007 to 30 May 2010

<table>
<thead>
<tr>
<th></th>
<th>DORIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>X pole mean</td>
<td>96 µas</td>
</tr>
<tr>
<td>Y pole mean</td>
<td>74 µas</td>
</tr>
<tr>
<td>X pole WRMS</td>
<td>650 µas</td>
</tr>
<tr>
<td>Y pole WRMS</td>
<td>519 µas</td>
</tr>
</tbody>
</table>

Station Coordinates

VLBI DORIS GPS Combination

Solutions by inversion of weekly unconstrained combined GPS SLR DORIS VLBI NEQ
Pole UT & Nutation are fixed to the a priori EOP C04 series
Weekly Terrestrial Station Coordinates are obtained with
Systematic Constraints: \( \sigma(Sx,Sy,Sz) = 1m \)
Minimal Constraints
Ties Constraints on 26 co-located sites

Site Ny-Alesund: 3 space geodetic techniques co-located, VLBI, GPS, DORIS

<table>
<thead>
<tr>
<th></th>
<th>( \bar{X} ) / mm</th>
<th>( \bar{Y} ) / mm</th>
<th>( \bar{Z} ) / mm</th>
<th>( \langle (X - \bar{X})^2 \rangle / mm )</th>
<th>( \langle (Y - \bar{Y})^2 \rangle / mm )</th>
<th>( \langle (Z - \bar{Z})^2 \rangle / mm )</th>
<th>( d\bar{X} ) / mm/y</th>
<th>( d\bar{Y} ) / mm/y</th>
<th>( d\bar{Z} ) / mm/y</th>
</tr>
</thead>
<tbody>
<tr>
<td>VLBI</td>
<td>+2.0</td>
<td>+25.3</td>
<td>-1.1</td>
<td>7.9</td>
<td>14.7</td>
<td>22.2</td>
<td>-2.3</td>
<td>+19.5</td>
<td>+52.5</td>
</tr>
<tr>
<td>GPS</td>
<td>+18.8</td>
<td>+11.8</td>
<td>+15.4</td>
<td>2.5</td>
<td>4.2</td>
<td>6.9</td>
<td>-4.0</td>
<td>+5.1</td>
<td>-4.9</td>
</tr>
<tr>
<td>DORIS</td>
<td>-2.2</td>
<td>+8.2</td>
<td>+6.9</td>
<td>8.1</td>
<td>7.0</td>
<td>11.4</td>
<td>+1.5</td>
<td>+6.7</td>
<td>+13.2</td>
</tr>
</tbody>
</table>

Transformation Parameters for GPS DORIS VLBI SLR and for GLOBAL combination with respect to ITRF2008

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DORIS Mean / σ</th>
<th>COMB Mean / σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx cm</td>
<td>+3.9 / 0.8</td>
<td>+5.9 / 1.1</td>
</tr>
<tr>
<td>Ty cm</td>
<td>-0.6 / 0.5</td>
<td>-1.1 / 0.4</td>
</tr>
<tr>
<td>Tz cm</td>
<td>-2.9 / 0.8</td>
<td>-2.1 / 0.7</td>
</tr>
<tr>
<td>D cm</td>
<td>-0.99 / 0.29</td>
<td>-0.38 / 0.24</td>
</tr>
<tr>
<td>Rx µas</td>
<td>+58.8 / 144</td>
<td>545 / 176</td>
</tr>
<tr>
<td>Ry µas</td>
<td>-75.3 / 153</td>
<td>199 / 171</td>
</tr>
<tr>
<td>Rz µas</td>
<td>+507 / 208</td>
<td>119 / 237</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DORIS Rate / σ</th>
<th>COMB Rate / σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx cm/y</td>
<td>+1.3 / 0.1</td>
<td>+1.7 / 0.2</td>
</tr>
<tr>
<td>Ty cm/y</td>
<td>-0.4 / 0.1</td>
<td>-0.5 / 0.1</td>
</tr>
<tr>
<td>Tz cm/y</td>
<td>-1.3 / 0.8</td>
<td>-1.1 / 0.1</td>
</tr>
<tr>
<td>D cm/y</td>
<td>-0.05/ 0.10</td>
<td>0.20 / 0.07</td>
</tr>
<tr>
<td>Rx µas/y</td>
<td>156 / 39</td>
<td>62 / 59</td>
</tr>
<tr>
<td>Ry µas/y</td>
<td>169 / 40</td>
<td>17 / 59</td>
</tr>
<tr>
<td>Rz µas/y</td>
<td>213 / 58</td>
<td>126 / 78</td>
</tr>
</tbody>
</table>

COL-WG Prospect

• **Re-iterate the CONT08 campaign** with homogenized standards and parameters, a priori reference system ITRF2008, ICRF2, EOP-C04, IERS conventions 2010

• **NEq multi-techniques combination** weekly bases on CONT08 campaign for several participating ACs have to be compared, multi-technique combination from different ACs have to be performed

• **New sets of data:** LEO satellites  
  Jason-2 (SLR, DORIS, GPS multi-techniques on board)  
  GRACE (SLR, GPS two techniques on board)

• **Study the sub-diurnal EOP** variations by deriving hourly or 2-hours estimates

• **Interpolation method** of a-priori EOPC04 for the data epoch has to be adopted

• The next **participation meetings**
  - 25th general assembly of the IUGG (28 June - 07 July 2011)
  - 3rd international colloquium “Scientific and Fundamental Aspects of the Galileo Program” (31 August 02 September 2011)
  - “Journées de référence spatio-temporel”s Paris September 2011
  - 3rd COL-WG meeting will be held at Paris (October 2011)

• **GRASP** space mission: “Geodetic Reference Antenna in Space” NASA’s Jet Propulsion Laboratory project will carry precise sensors system for GNSS, SLR, DORIS and VLBI geodetic techniques on space board satellite. The launch is expected in 2017.

• **HY-2A** satellite mission using the SLR GPS and DORIS technique on board
Thanks for your attention