DORIS DAYS 2025

November, 3 and 5

DORIS observation equation

Jean-Michel Lemoine (CNES)

November 5

Session 2 – Presentation 3

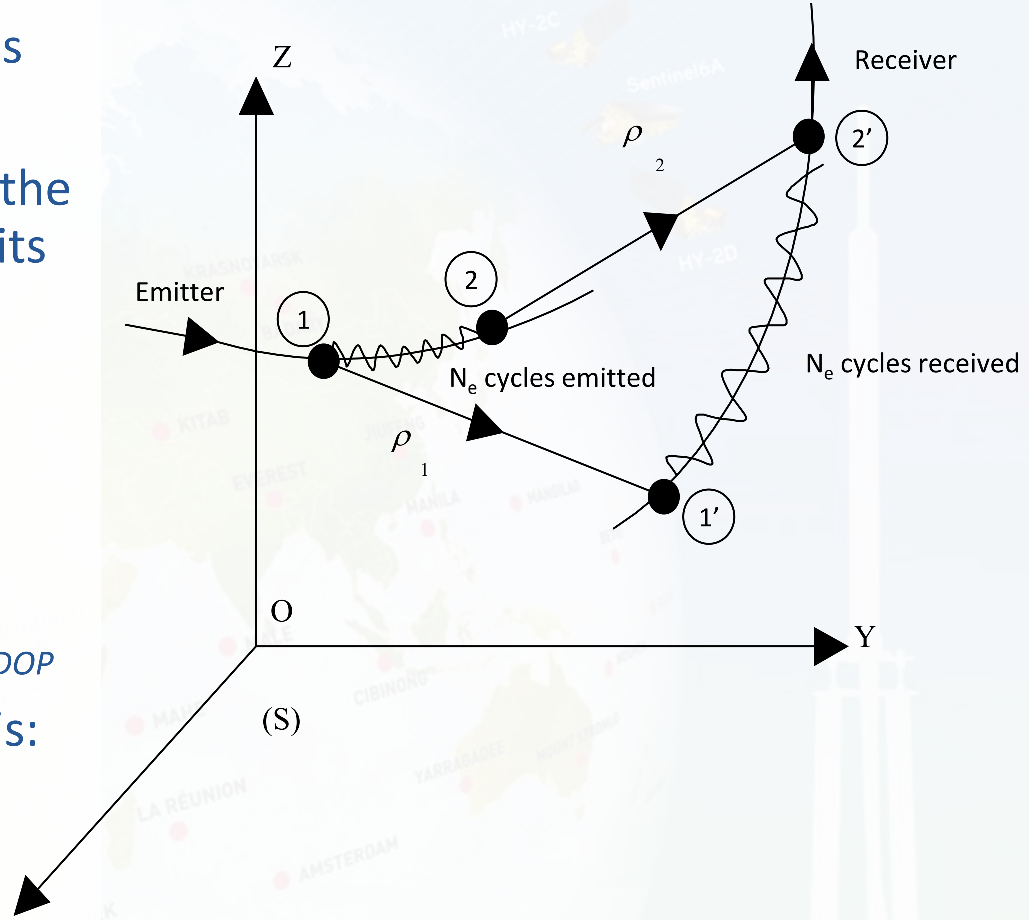
Summary

- The Doppler measurement
- Corrections to the doppler measurement:
  - Time-tagging of the measurements in the TAI scale
  - Troposphere
  - Ionosphere (and iono-free phase centers)
  - Light-time correction
  - Frequency offsets
  - South Atlantic Anomaly perturbations
  - Phase Wind-Up
  - Relativity

The DORIS phase measurement

- Between events ❶ and ❷, N_e cycles are emitted by the DORIS beacon at frequency f_e
- This radio electric signal travels through space and reaches the receiver at events ❶' and ❷'
- During the time separating events ❶' and ❷' on-board the satellite, the DORIS receiver has generated N_r cycles with its proper oscillator at frequency f_r
- The DORIS receiver then accurately measures the phase difference between the received and generated signals, every x (3 or 7 or 10) seconds (Δt)
- This measurement can be used as an ambiguous phase measurement or, by differentiation, as a Doppler count N_{DOP}
- In that case, the simplified velocity observation equation is:

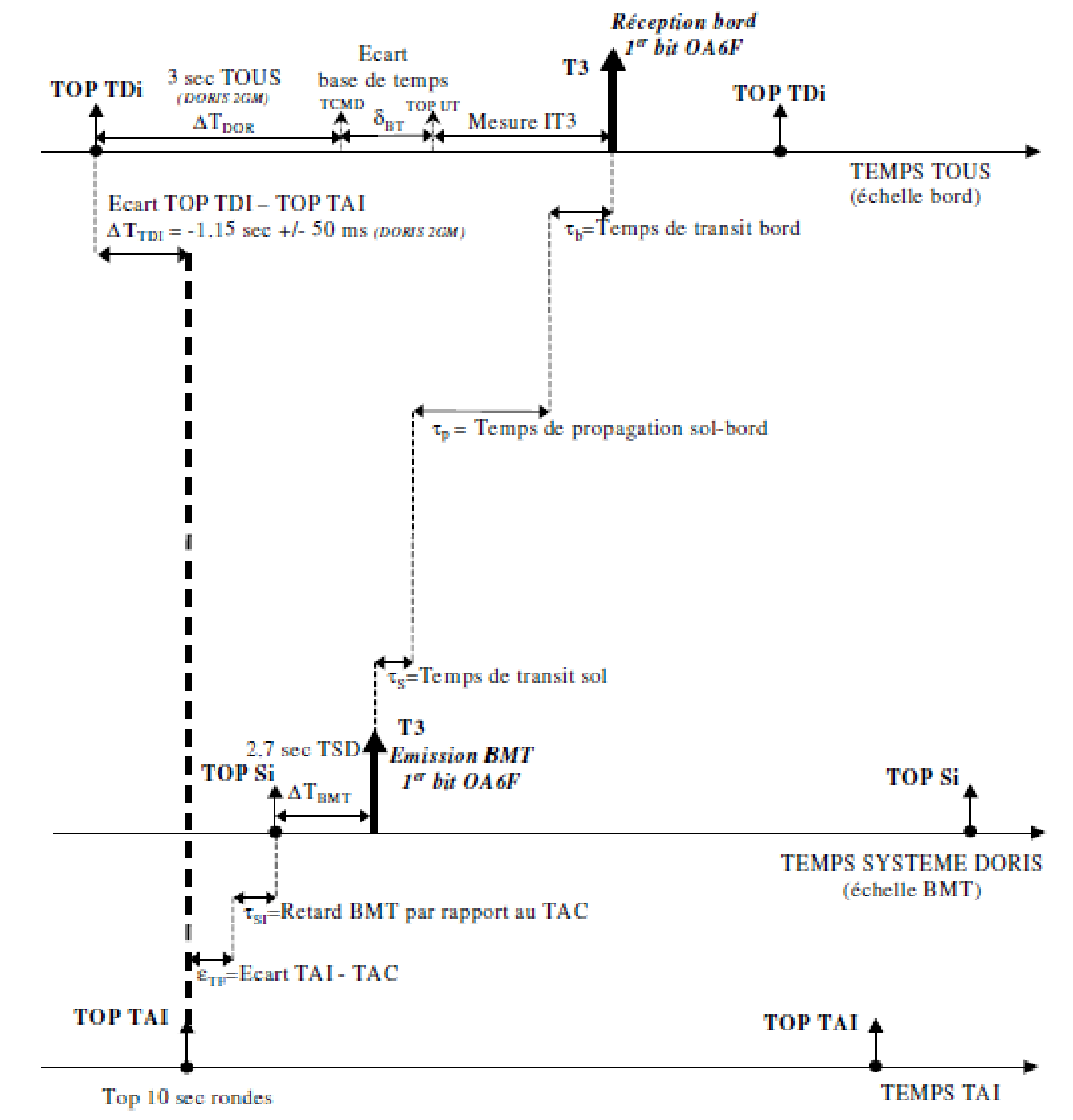
$$V_{E\_R} = \frac{c}{f_e}\left(f_e - f_r - \frac{N_{Dop}}{\Delta t}\right)$$



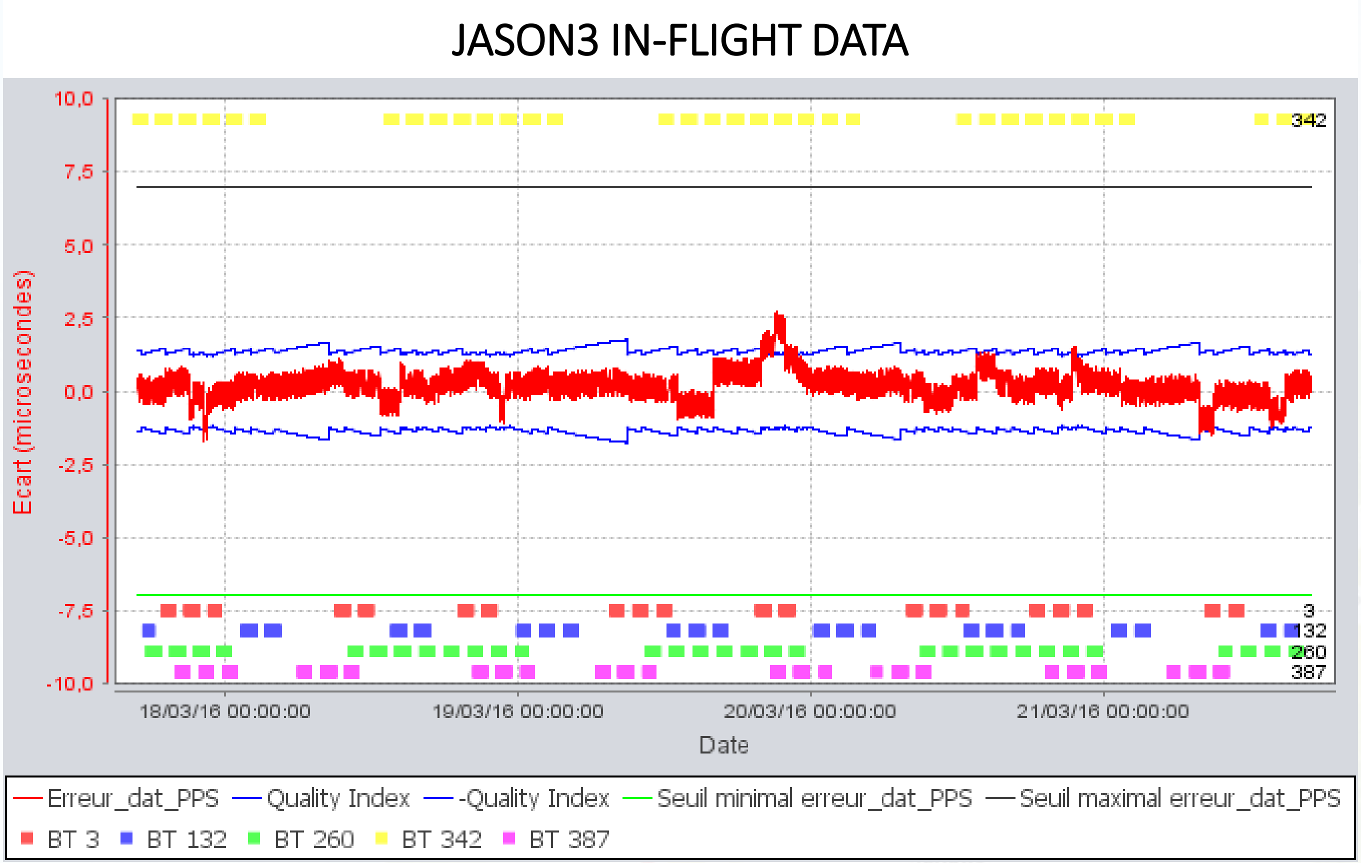
Lemoine J-M, Capdeville H, Soudarin L (2016) Precise orbit determination and station position estimation using DORIS RINEX data. Advances in Space Research 58:2677–2690. https://doi.org/10.1016/j.asr.2016.06.024

On-board time tagging

- Each ground beacon emits, every 10 s, a synchronisation bit “IT3”, which is received on-board and compared with the receiver time scale
- The bits received from the Time Beacons are then used to compute the on-board time scale offset
- Individual accuracy : 7 microseconds on 400 MHz channel
- Accuracy after post-processing: 1 -2 microseconds



DORIS/Diode Time-tagging accuracy



Accuracy : 1 - 2 microseconds

(which is equivalent to 7 – 14 mm Along-Track & 1 – 2 millimetres Radial)

Time Beacons :

- Toulouse (France)
- Kourou (French Guyana)
- Hartebeesthoek (South Africa)
- Papeete (French Polynesia)
- (Yellowknife, Canada)
- Terre Adélie (Antarctica)

DORIS RINEX measurement content

```
> 2001 08 21 00 00 39.939956370  0  2        -1.084696938 0
```

YEAR | MONTH | DAY | HOUR | MINUTE | SECOND in DORIS timescale | OFFSET between DORIS timescale and TAI from DIODE

Figure 1: Example of a DORIS/RINEX epoch record

Ambiguous phase on Freq. 1 | Ambiguous phase on Freq. 2 | Pseudo-range from Freq. 1 | Pseudo-range from Freq. 2

```
D01  -1907631.062 1   -375988.691 1 32743488.281 1 32743301.603 1     -130.250 7
         -116.250 7      2361.256       1000.820 1        0.000 1       72.732 1
D02        -0.000 1        -0.000 1 32884249.705 2 32884916.645 2     -139.000 7
         -126.400 7      2361.256       1000.773 1       16.628 1       72.738 1
```

Relative frequency offset of the Receiver | Pressure | Temperature | Humidity

Figure 2: Example of two DORIS/RINEX observation records

```
G    5 C1P L1P L2C C2C S2C                                SYS / # / OBS TYPES
R    2 C1C L1C                                            SYS / # / OBS TYPES
E    2 L1B L5I                                            SYS / # / OBS TYPES
S    2 C1C L1C                                            SYS / # / OBS TYPES
D   10  L1  L2  C1  C2  W1  W2   F   P   T   H            SYS / # / OBS TYPES
```

Figure 3: Line of the DORIS/RINEX file header (in grey) describing the 10 DORIS observation fields

Measurement corrections

- **Ionospheric correction**: the first order correction is obtained thanks to the dual frequencies f_2 and f_{400}
- **Tropospheric correction**: the dry and wet parts are computed either from meteorological data collected at the station, or from models (e.g. VMF-1). In any case, an additional tropospheric zenithal bias has to be adjusted at each pass (at least)
- **Frequency offsets** of the beacon and receiver oscillator: an empirical frequency bias has to be adjusted at each pass
- **Phase Wind-Up:** comes from the respective rotations of the emitting and receiving antennae
- **Relativity**: the effect of relativity on the beacon and receiver clock oscillators has to be taken into account, as well the elongation of the travel time of the RF signal

Tropospheric correction

Refer to Ejo Schrama's presentation

Time-of-flight correction

Refer to Ejo Schrama's presentation

Ionospheric correction

Refer also to Ejo Schrama's presentation, and

- **Ionospheric correction**: the first order correction is obtained thanks to the dual frequencies f_2 and f_{400} , where $L_{iono\text{-}free\text{-}2GHz}$ is the iono-free phase measurement on 2 GHz:

$$L_{iono-free-2GHz} = L_{2GHz} + \frac{L_{2GHz} - \sqrt{\gamma} L_{400MHz}}{\gamma - 1} \quad \text{with} \quad \gamma = \left(f_{2GHz} / f_{400MHz} \right)^2$$

- **CAUTION**: to this iono-free measurement are associated iono-free phase centres:

$$\vec{r}_{2GHz,iono-free} = \frac{\vec{r}_{400\,MHz,2GHz}}{\gamma - 1} \qquad \textbf{(20)}$$

Where $\vec{r}_{2GHz,iono-free}$ is the vector from the 2 GHz phase center to the iono-free phase center and $\vec{r}_{400\,MHz,2GHz}$ is the vector from the 400 MHz to the 2 GHz phase center.

In the case of DORIS, the nominal frequencies are $f_{2GHz} = 2.036250$ GHz, $f_{400\,MHz} = 401.250$ MHz, therefore $\gamma = 25.75325356$ and the iono-free phase centers are located a few mm away from the 2 GHz phase centers, in the direction opposite to the 400 MHz phase centers.

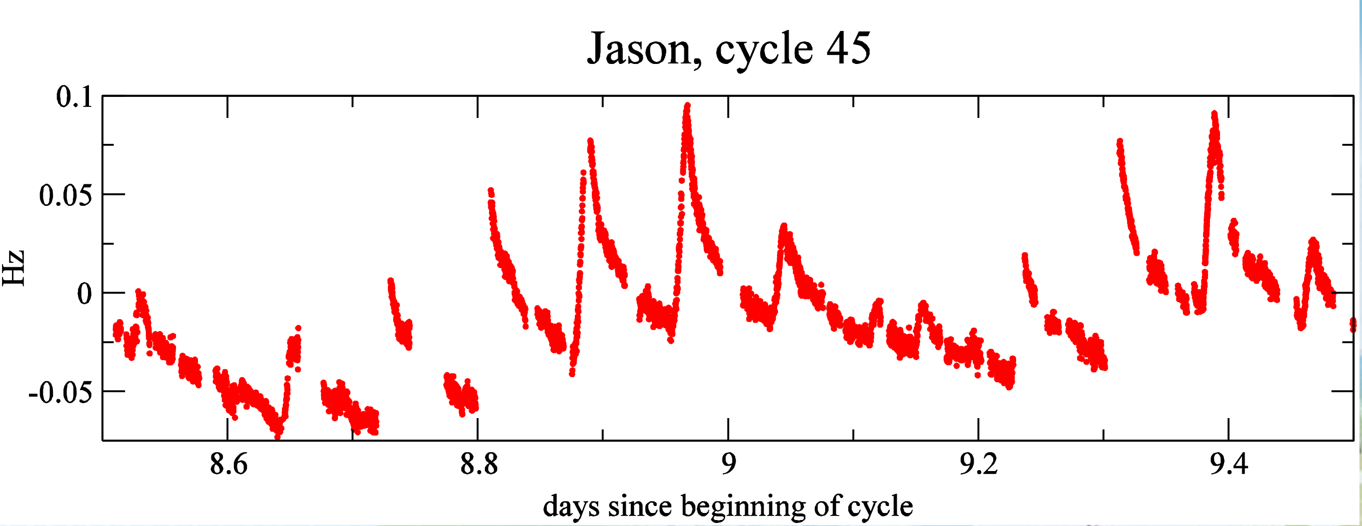
Frequency offsets

- The simplified equation $V_{e-r} = \frac{c}{f_e}\left(f_e - f_r - \frac{N_{DOP}}{\Delta t}\right)$ assumes the emitter and receiver frequencies are nominal
- In fact it must be re-written $V_{e-r} = \frac{c}{f_e + \Delta f_e}\left(f_e + \Delta f_e - (f_r + \Delta f_r) - \frac{N_{DOP}}{\Delta t}\right)$

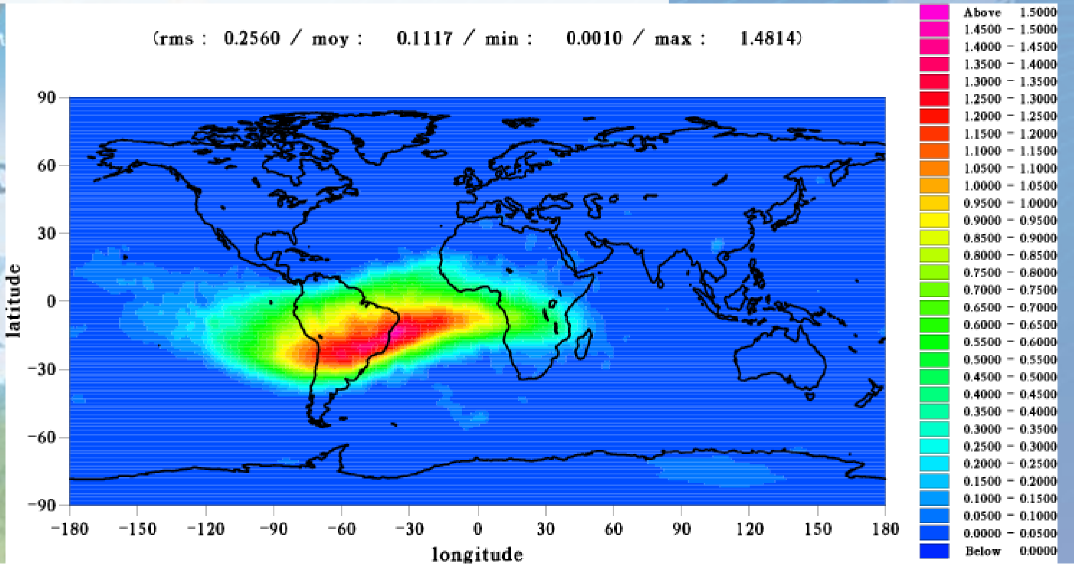
Where Δf_e and Δf_r are emitter and receiver frequency offsets.

- In practice, Δf_e and Δf_r cannot be determined independently. An approximate value of Δf_r is provided by DIODE, thus only Δf_e is adjusted by least squares during the orbit adjustment process.

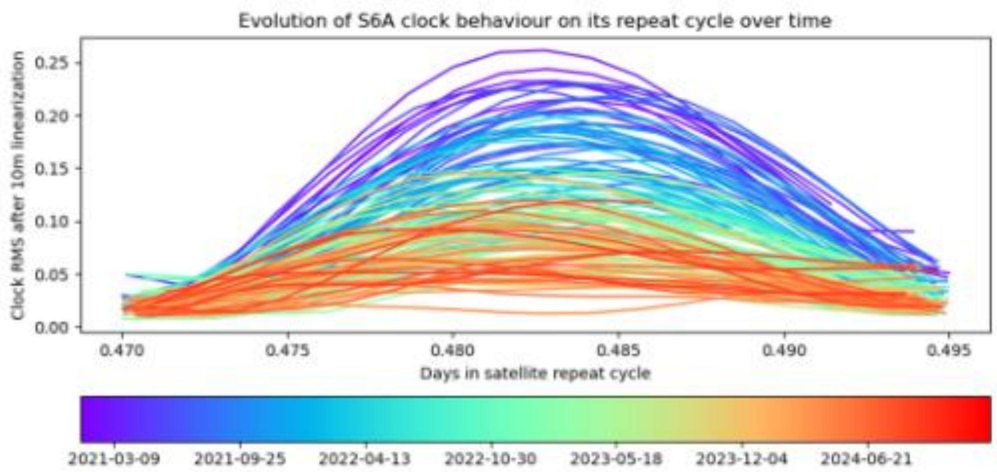
South Atlantic Anomaly perturbations



Jason-1 receiver frequency offsets



Plot of the relative SAA dose exposure in 2002-2005 at the 1300 km altitude of Jason-1



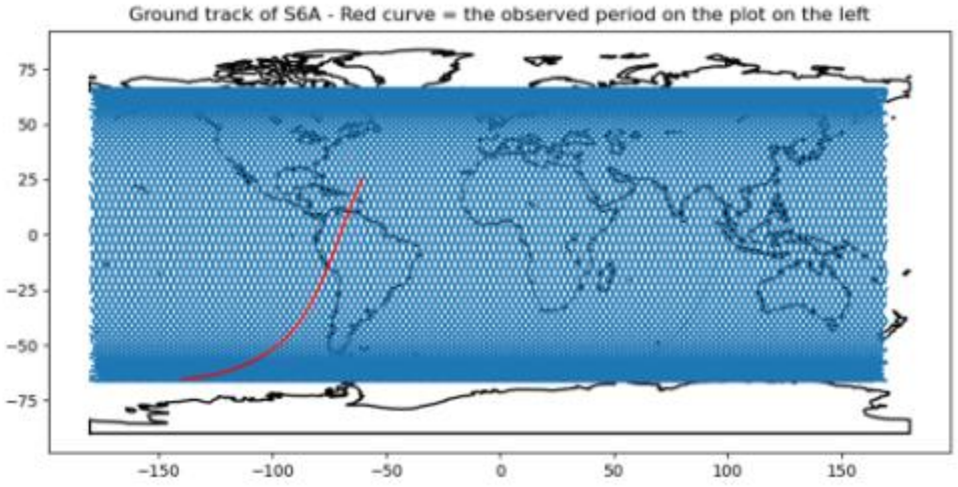
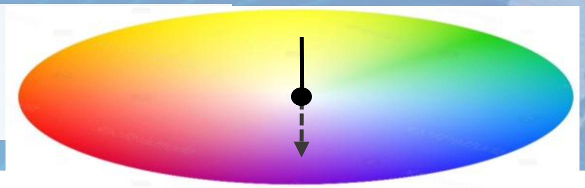
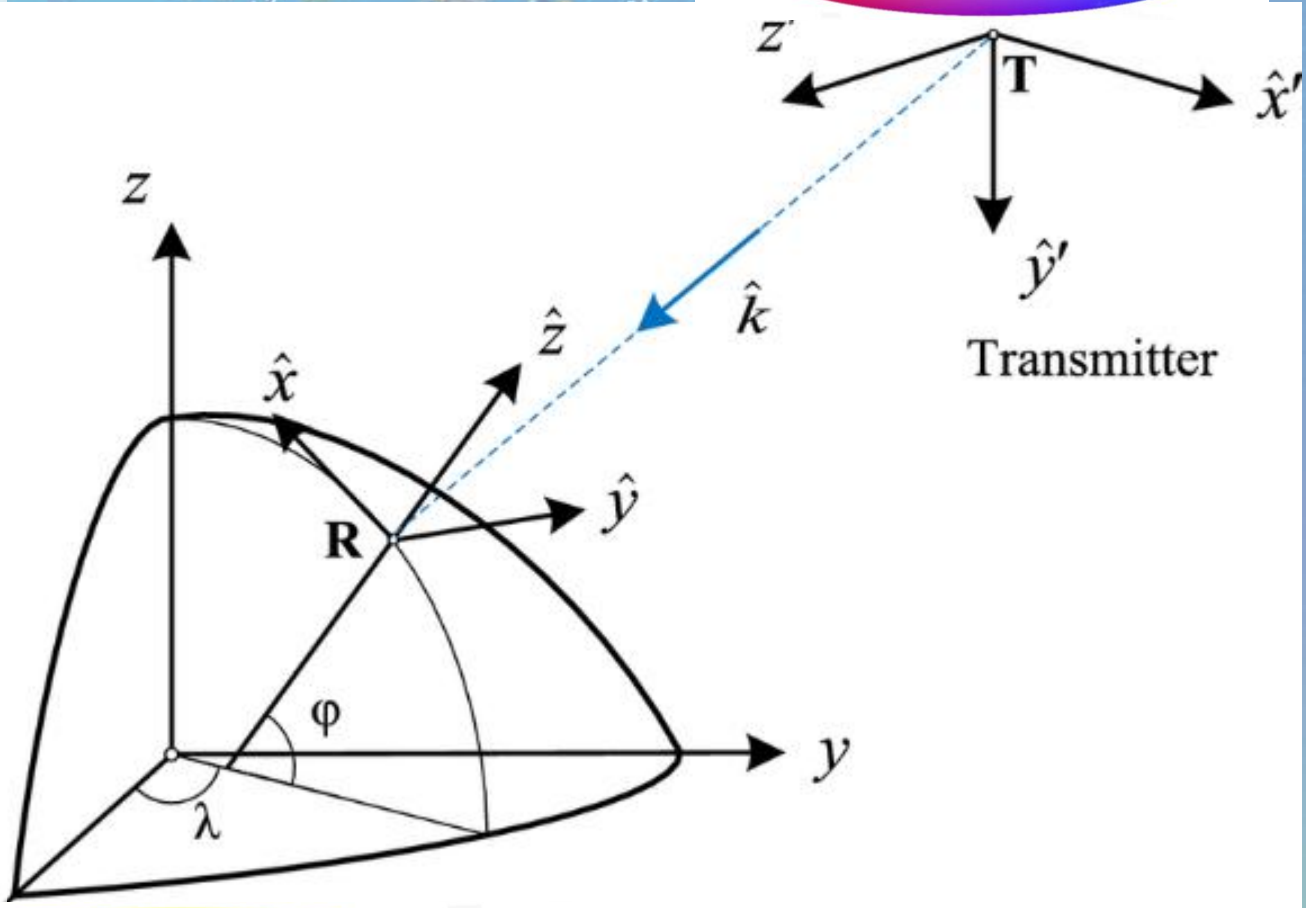


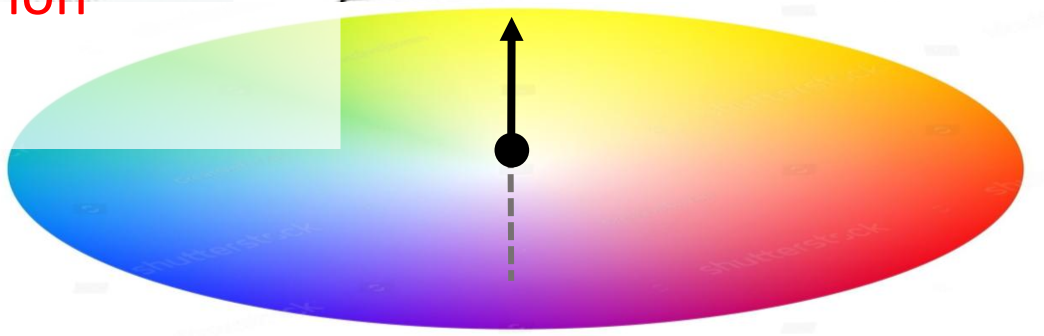
FIGURE : Aging of the SAA effect for Sentinel-6 MF responsible for station height secular trends.

Phase Wind-Up

- The **DORIS** Phase Wind-Up effect (PWU) is similar to the **GPS** PWU
- It comes from the fact that the phase of the emitter antenna as well as receiver antenna are circular and perform a 360° turn around the antenna axis
- The emitter antenna is pointing upwards, the receiver one downwards
- If the trajectory between emitter is linear, the PWU almost cancel out
- However, if the satellite is performing an attitude manoeuvre, it will be non negligible





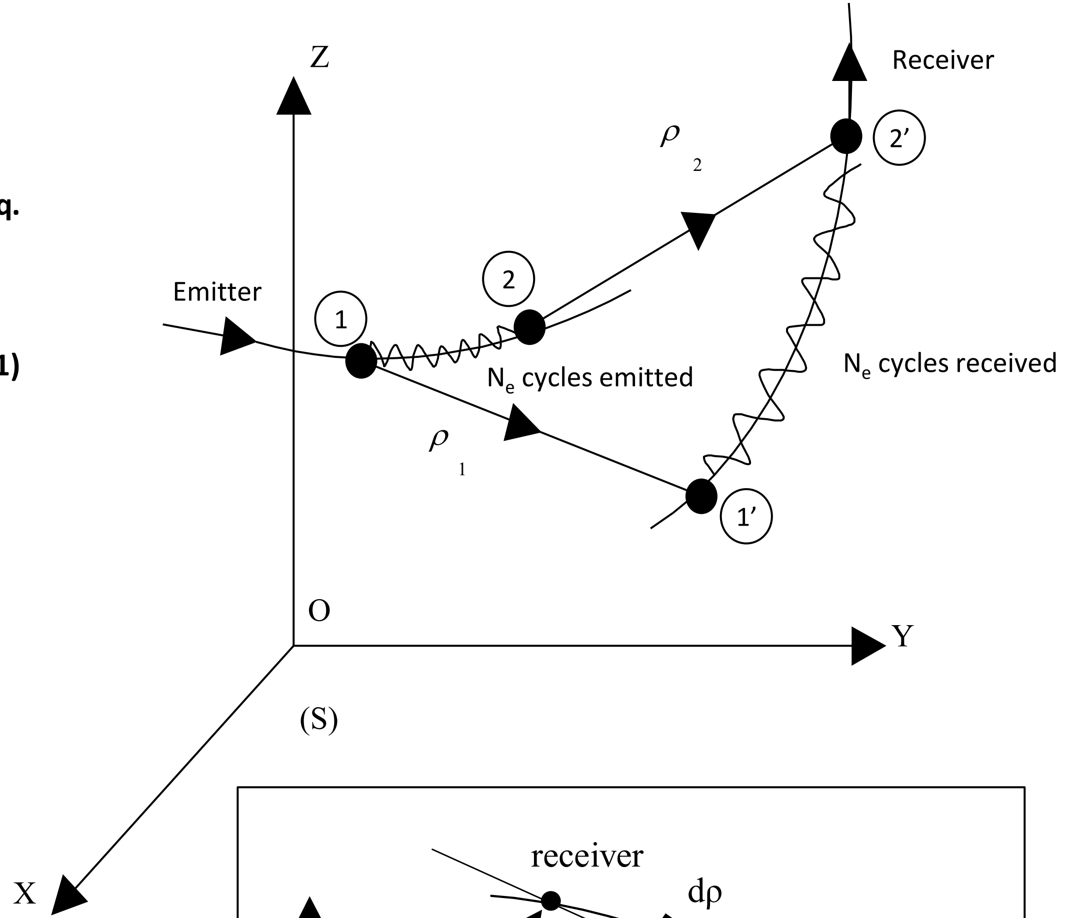


Relativity

The conversion between the proper time τ of a clock and the coordinate time t is approximated by **Eq. (1)** (e.g. Eq. 6 in Moyer, 1981 or Eq. 10.7 in Petit and Luzum, 2010):

$$d\tau \approx \left[1 - \frac{U}{c^2} - \frac{V^2}{2c^2}\right] dt \qquad \textbf{(1)}$$

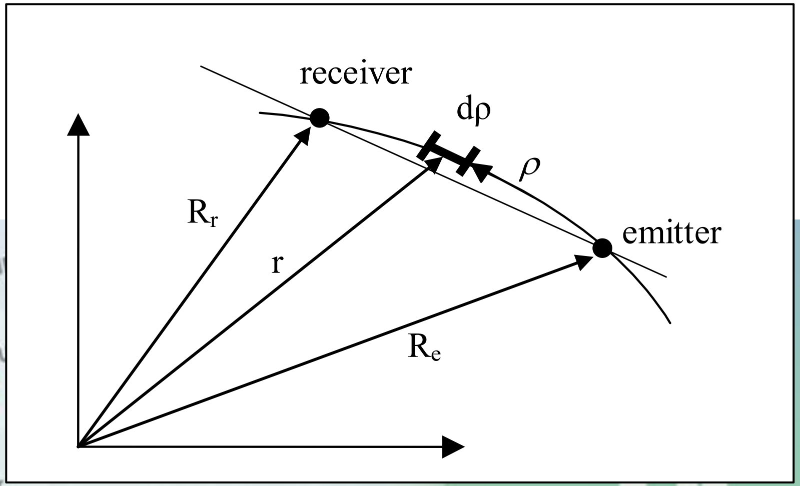
Where:
U is the gravitational potential at the location of the clock;
V is the velocity of the clock in the coordinate reference frame;
c is the velocity of light in the vacuum.



The travel time between the emitter and the receiver in the vacuum can be approximated by **Eq. (2)**:

$$\Delta t_{travel} \approx \frac{\rho}{c} + 2\frac{\mu}{c^3} \ln\left(\frac{R_e + R_r + \rho}{R_e + R_r - \rho}\right) \qquad \textbf{(2)}$$

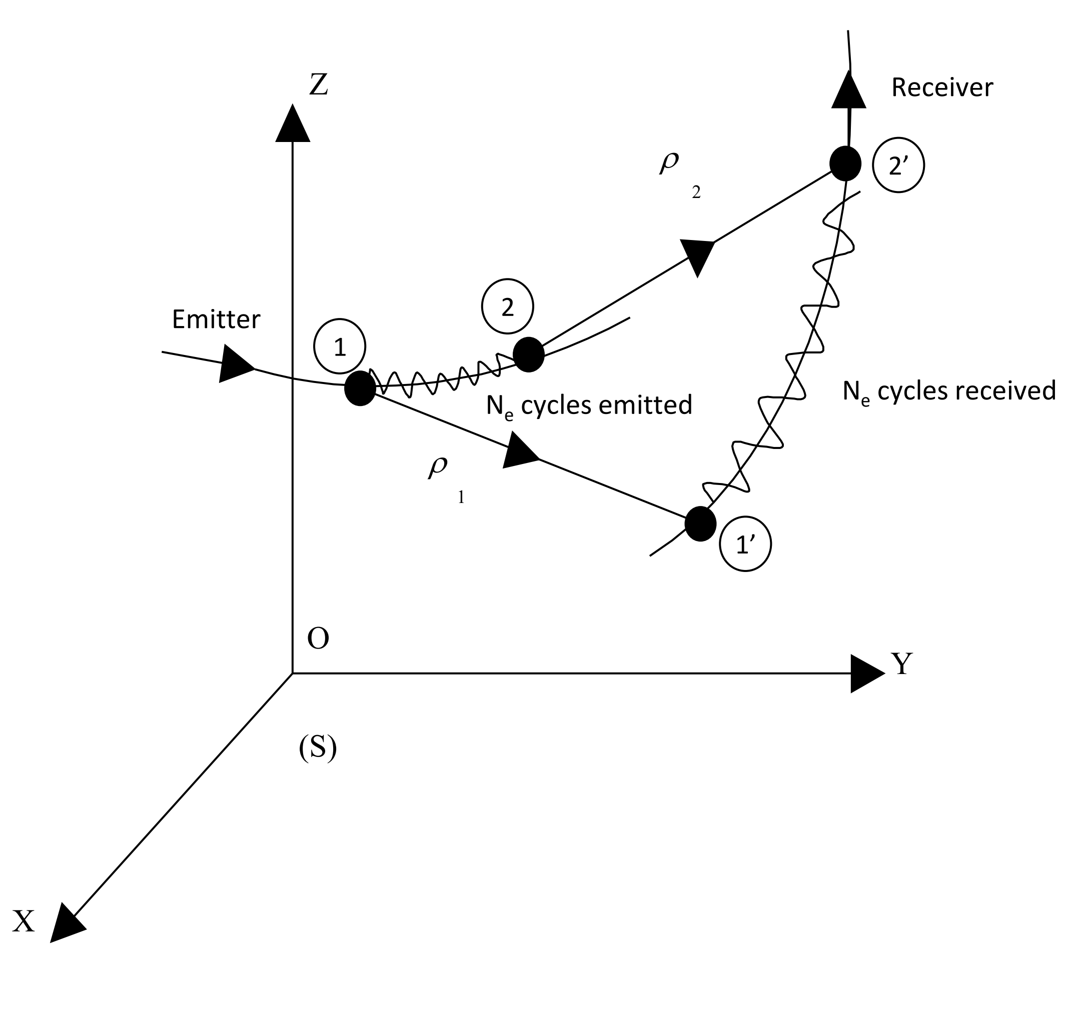
Where:
The subscript e denotes the emitter and r the receiver;
ρ is the curvilinear trajectory of the photon; close at the first order to the geometrical distance between the emitter and the receiver;
$\mu = GM$, with G the gravitational constant, M the mass of the Earth;
R_e and R_r are the geometrical distance of the emitter (resp. receiver) to the center of the reference frame, coincident with (or close to) the center of mass of the Earth.



Relativity

1. $\Delta v_{REL} = \Delta v_{REL_C} + \Delta v_{REL_T}$ is the relativistic correction which is composed of two parts, the clock correction Δv_{REL_C} and the travel correction Δv_{REL_T} :

$$\begin{cases} \Delta v_{REL_C} = \dfrac{1}{c}\left[U_r - U_e + \dfrac{V_r^2 - V_e^2}{2}\right] \\ \Delta v_{REL_T} = \dfrac{2\mu}{\Delta\tau_r\, c^2}\left[\ln\left(\dfrac{R_1 + R_{1'} + \rho_1}{R_1 + R_{1'} - \rho_1}\right) - \ln\left(\dfrac{R_2 + R_{2'} + \rho_2}{R_2 + R_{2'} - \rho_2}\right)\right] \end{cases}$$



The end

- We have gone quickly through the DORIS Doppler observation itself and most of its main corrections