DORIS DAYS 2025

November, 3 and 5

Atmospheric Effects

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Session 2 – Presentation 2

IDS

Atmospheric Effects on DORIS Doppler Measurements

Objectives:
⇨ how the DORIS receiver forms its Doppler measurement
⇨ how the optical path enters the equation
⇨ how the atmosphere affects signal propagation

Outline :

- Formation of the Doppler Observable
- Optical Path and Signal Propagation
- Atmospheric Structure and Physical Processes
- Tropospheric Modeling
- Ionospheric Modeling

General Principle of the DORIS System

- Ground beacons emit two continuous signals:

$$f_1 = 2.03625\ \text{GHz}, \qquad f_2 = 401.25\ \text{MHz}.$$

- The satellite compares the received frequency f_r with the nominal frequency f_0.
- Over a 10-second integration interval (T = 10 s) :

$$N_{\text{obs}} = \int_{t_0}^{t_0+T} (f_r - f_0)\, dt.$$

⇨ **Number of Doppler cycles accumulated** during the measurement.

Ultra-Stable Oscillator (USO)

- The USO provides a highly stable on-board frequency reference:

$$f_{\mathrm{USO}} = 10\ \mathrm{MHz}.$$

- Counting 10^8 cycles of this reference **defines T,** the **10-second integration interval** (USO time).

- During each 10-s integration, the receiver compares the **measured** number of received cycles with the **expected** number:

$$N_{\mathrm{obs}} = N_{\mathrm{rec}} - f_0 T$$

- The Doppler observable is therefore expressed in **cycles accumulated over the 10-second interval**, which can later be converted into a **mean frequency shift**.

From Cycle Count to Mean Frequency

- Starting from the integrated observable:

$$N_{\mathrm{obs}} = \int_{t_0}^{t_0+T} (f_r - f_0)\, dt$$

- The mean frequency shift over the 10-s window is:

$$\boxed{\bar{D} = \frac{N_{\mathrm{obs}}}{T} = \bar{f}_r - f_0.}$$

⇨ The Doppler observable originates from a **cycle count**, then normalized by the integration time to yield an **average frequency difference (Hz)**.

Time Notation and Propagation Equation

Let :

- t_e : emission time at the beacon,
- t_r : reception time at the satellite (same time scale, e.g. TT or TAI).

The propagation (light-time) relation is:

$$\boxed{t_r = t_e + T_{\text{prop}}(t_e)}, \qquad T_{\text{prop}} = \frac{L(t_e)}{c}.$$

where $L(t_e)$ is the **optical path length**, including all physical propagation effects (geometric, atmospheric, relativistic, instrumental).

Phase Conservation and Signal Propagation

- Phase continuity along the ray: $$\phi_r(t_r) = \phi_e(t_e).$$

- Differentiating with respect to t_r : $$\frac{d\phi_r}{dt_r} = \frac{d\phi_e}{dt_e}\frac{dt_e}{dt_r}.$$
  (with $t_e = t_e(t_r)$)

- Since the time derivative of phase gives angular frequency $(\omega = 2\pi f)$:

$$2\pi f_r = 2\pi f_e \frac{dt_e}{dt_r} \quad \Rightarrow \quad \boxed{f_r = f_e \frac{dt_e}{dt_r}}.$$

- The propagation (light-time) relation provides the key link between phase continuity and the Doppler relation.

Proof Sketch: Derivation of the Doppler Equation

From the propagation (light-time) relation :

$$t_r = t_e + \frac{L(t_e)}{c},$$

differentiate w.r.t. t_r :
(with $t_e = t_e(t_r)$)

$$1 = \frac{dt_e}{dt_r}\left(1 + \frac{1}{c}\frac{dL}{dt_e}\right) \Rightarrow \boxed{\frac{dt_e}{dt_r} = \frac{1}{1 + \frac{1}{c}\frac{dL}{dt_e}}}.$$

Substitute into $f_r = f_e \frac{dt_e}{dt_r}$:

$$\boxed{f_r = \frac{f_e}{1 + \frac{1}{c}\frac{dL}{dt_e}}} \simeq f_e\left(1 - \frac{1}{c}\frac{dL}{dt_e}\right)$$

Thus the relative Doppler (1st order):

$$\boxed{\frac{f_r - f_e}{f_e} = -\frac{1}{c}\frac{dL}{dt_e}}.$$

Doppler and Optical Path Variation

$$\frac{f_r - f_e}{f_e} = -\frac{1}{c}\frac{dL}{dt_e}.$$

When we measure the Doppler, we're **not directly measuring the transmitter's frequency**.

We're measuring **how the distance between the beacon and the satellite changes over time** : their **relative velocity along the line of sight.**

$$\dot{L} = \frac{dL}{dt_e}$$

⇨ The Doppler observable thus represents the **rate of change of the optical path**, interpreted as an **apparent radial velocity**, not as a frequency itself.

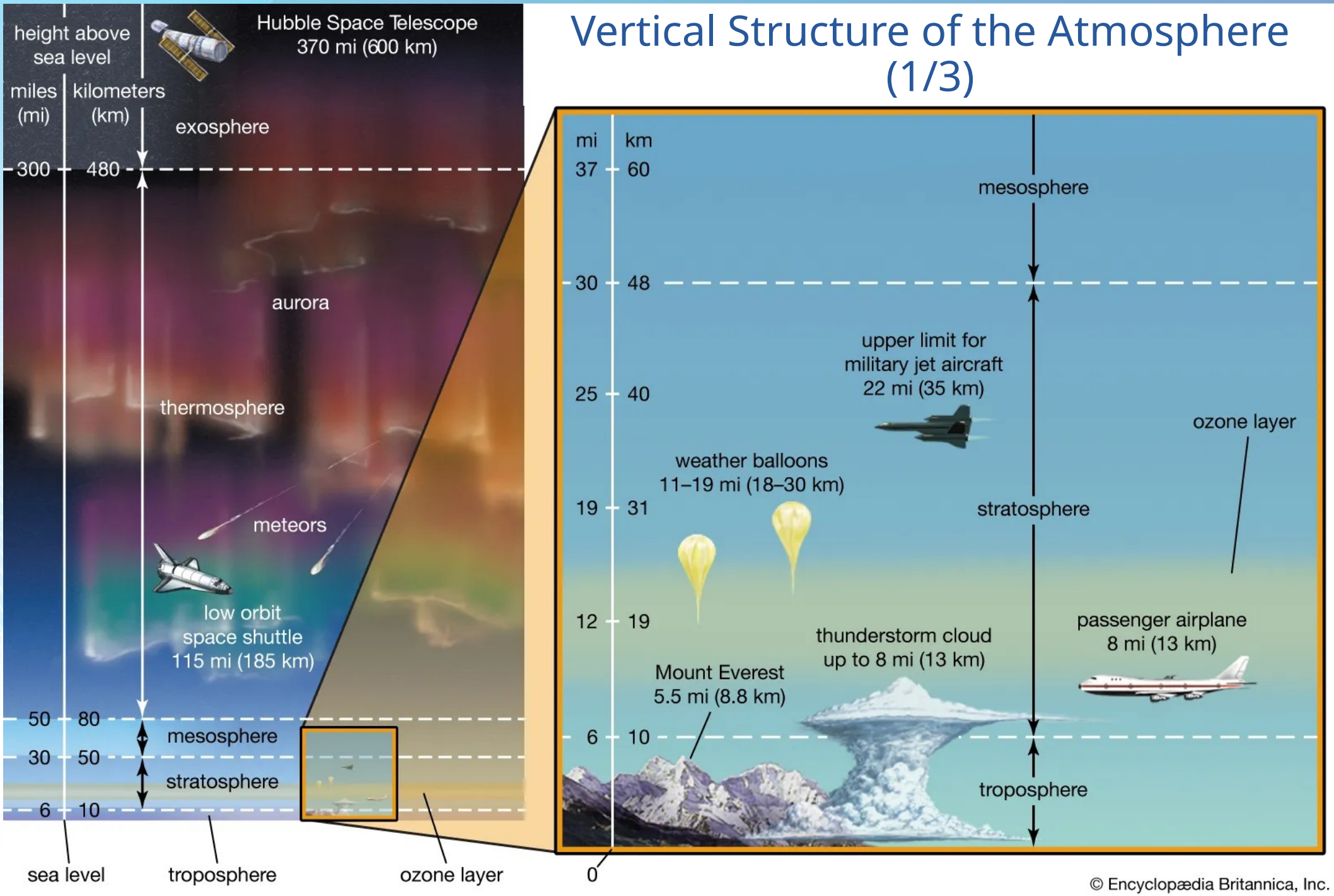
Complete Definition of the Optical Path

$$L = \rho + L_{\text{atm}} + L_{\text{rel}} + L_{\text{inst}}.$$

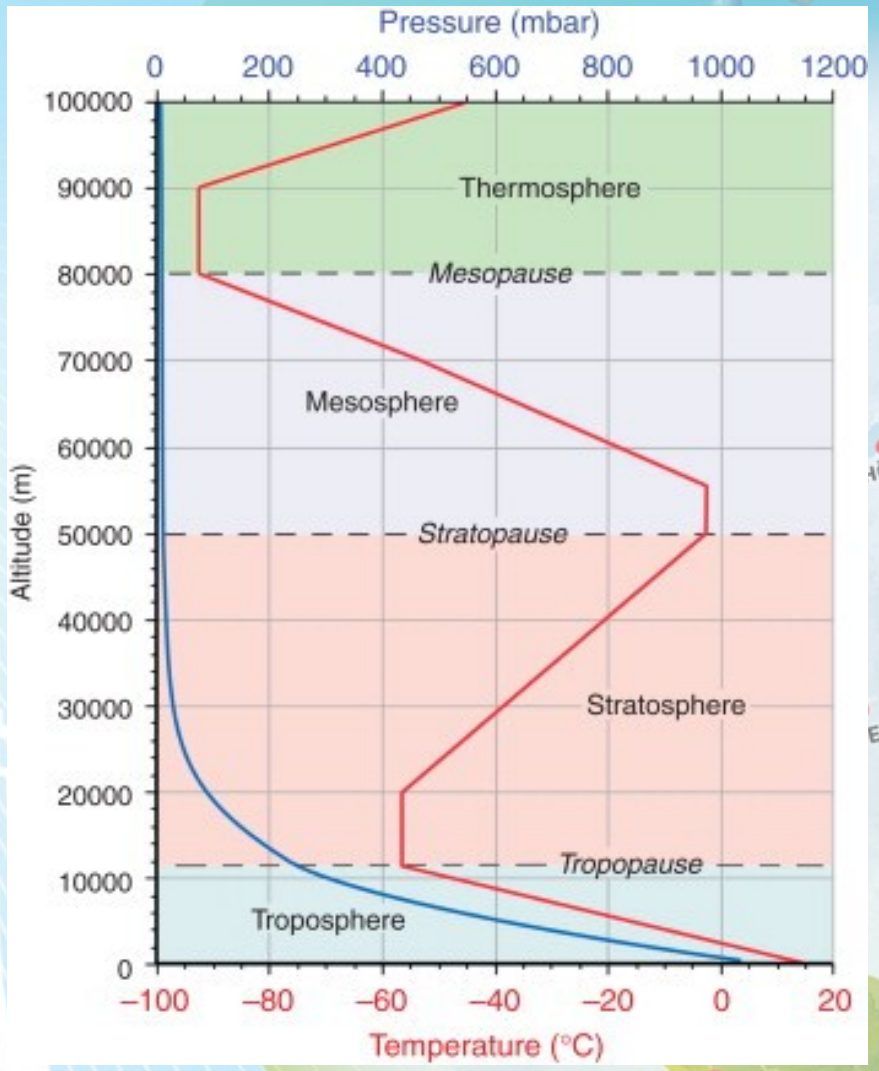
- ρ : geometric distance $|\mathbf{r}_s - \mathbf{r}_b|$
- L_{atm} : atmospheric contribution (troposphere + ionosphere)
- L_{rel} : relativistic effects (Sagnac, Shapiro, etc.)
- L_{inst} : instrumental delays

In this lecture, we focus on L_{atm}.

Vertical Structure of the Atmosphere (1/3)



Vertical Structure of the Atmosphere (2/3)

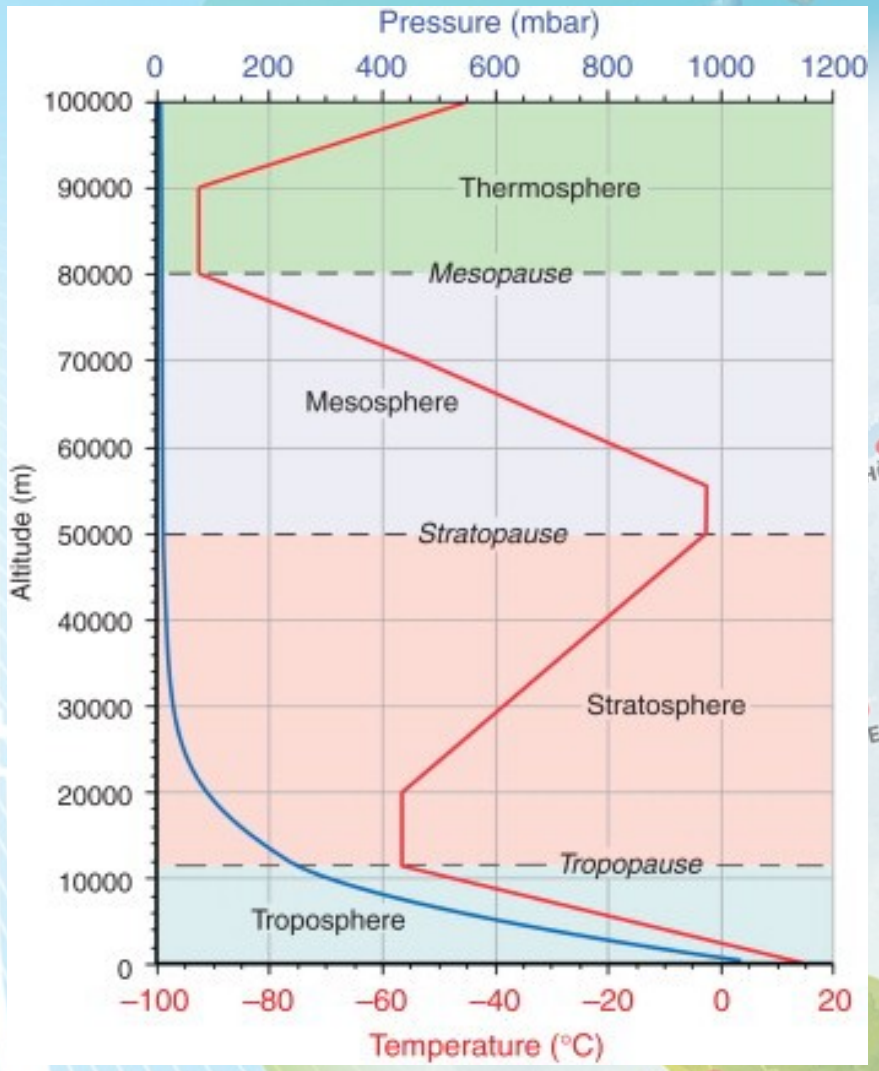


[www.physicalgeography.net/fundamentals/7b.html]

The vertical **temperature profile** indicating the layers of the atmosphere (USSA 1976):

- troposphere : lower (0–11 km) atmosphere
- stratosphere: lower-middle (20–50 km) atmosphere,
- mesosphere : upper-middle (56–80 km) atmosphere,
- thermosphere : upper (90–100 km) atmosphere,

Vertical Structure of the Atmosphere (3/3)



- The vertical **temperature profile** indicating the layers of the atmosphere (USSA 1976):

  The **turning points of the temperature profile** are formed by intermediate layers :

- the tropopause (11–20 km),
- the stratopause (50–56 km),
- and the mesopause (80–90 km).

[www.physicalgeography.net/fundamentals/7b.html]

Physical Processes in Atmospheric Layers (< 80 km)

Layer	Altitude (km)	Main physical processes	Impact on signal propagation
Troposphere	0 – 10	Convection and turbulent mixing of air heated by the surface. Strong vertical gradients of temperature, pressure, and water vapor.	Non-dispersive delay due to refractive index $n > 1$. Introduces ZHD + ZWD.
Stratosphere	10 – 50	Absorption of solar UV by ozone. Stable layer, no convection. Temperature increases with altitude.	Still non-dispersive; contributes mainly to ZHD.
Mesosphere	50 – 80	Absorption of radiation and weak convection. Temperature decreases again with altitude.	Very small effect; transition to ionized layers.

Physical Processes in Atmospheric Layers (> 80 km)

Layer	Altitude (km)	Main physical processes	Impact on signal propagation
Thermosphere	80 – 500	Strong heating by solar X-rays and γ-rays; molecules and atoms become partially ionized. Temperature increases sharply with altitude.	Beginning of the ionosphere; **dispersive delay** proportional to $1/f^2$.

- Above roughly 80–100 km, the **temperature** keeps increasing with altitude and **no longer defines distinct atmospheric layers**.
- Instead, the **degree of ionization**, expressed through the **electron density** Ne(h), becomes the dominant physical parameter describing the upper atmosphere.
- This region corresponds to the thermosphere, within which lies the ionosphere.

Ionosphere: Electron density structure and layers

- The ionosphere is stratified according to the electron density profile $N_e(h)$

$$\begin{cases} \text{D region: 60–90 km,} & N_e \approx 10^{8\text{–}9}\ \text{m}^{-3}, \text{present only during daytime;} \\ \text{E region: 90–130 km,} & N_e \approx 10^{10\text{–}11}\ \text{m}^{-3}, \text{strong by day, weak or residual at night;} \\ \text{F}_1 \text{ region: 150–200 km,} & N_e \approx 10^{11\text{–}12}\ \text{m}^{-3}, \text{daytime only;} \\ \text{F}_2 \text{ region: 250–500 km,} & N_e > 10^{12}\ \text{m}^{-3}, \text{persists through the night.} \end{cases}$$

- The **F_2 layer** contains most of the ionospheric electrons and is the main contributor to the **dispersive delay** affecting DORIS and other microwave systems.
- ⚠️ Scintillation : Small-scale irregularities in the electron density distort radio wavefronts → rapid phase and amplitude fluctuations, often leading to **loss of lock on the 401 MHz signal.**

Ionosphere: Day–Night variations

$$\begin{cases} \text{D region: 60–90 km,} & N_e \approx 10^{8\text{–}9}\ \mathrm{m}^{-3}, \text{present only during daytime;} \\ \text{E region: 90–130 km,} & N_e \approx 10^{10\text{–}11}\ \mathrm{m}^{-3}, \text{strong by day, weak or residual at night;} \\ \mathrm{F}_1 \text{ region: 150–200 km,} & N_e \approx 10^{11\text{–}12}\ \mathrm{m}^{-3}, \text{daytime only;} \\ \mathrm{F}_2 \text{ region: 250–500 km,} & N_e > 10^{12}\ \mathrm{m}^{-3}, \text{persists through the night.} \end{cases}$$

- **Daytime**: intense solar EUV and X-ray radiation causes strong ionization → all layers (D, E, F_1, F_2) exist.
  Electron density peaks near 300–400 km.

- **Nighttime**: ionization ceases → recombination dominates.
  The **D** and **F_1** regions **disappear**, the **E** region becomes **weak** or intermittent (sometimes maintained by auroral or sporadic events), and **F_2** remains as a **diffuse F layer**.

- As a result, the **Total Electron Content (TEC)** is much larger during the day and more stable at night.

Atmospheric Path in the Observation Model

We write the optical path as

$$L = \rho + L_{\text{trop}}(\epsilon) + L_{\text{ion}}(f,\epsilon) + L_{\text{rel}} + L_{\text{inst}}, \quad L_{\text{atm}} = L_{\text{trop}}(\epsilon) + L_{\text{ion}}(f,\epsilon)$$

- **Troposphere**: non-dispersive $\Rightarrow$ depends on elevation via mapping functions.
- **Ionosphere**: dispersive $\propto 1/f^2$ and geometry (path through N_e).
- **Doppler link**: $\dfrac{f_r - f_e}{f_e} = -\dfrac{1}{c}\dfrac{dL}{dt_e} \Rightarrow$ we need $\dot{L}_{\text{trop}}$ and $\dot{L}_{\text{ion}}$.

Next slides (operational targets):

$ZHD,\ ZWD,\ m_h(\epsilon),\ m_w(\epsilon)$ $\rightarrow L_{\text{trop}}(\epsilon) = ZHD\, m_h + ZWD\, m_w$

$L_{\text{ion}}(f) = 40.3\, TEC/f^2$ $\rightarrow$ iono-free combination.

*(Note: L_{ion} shown as **group delay**; phase delay has opposite sign but same $1/f^2$ law.)*

Troposphere: Refractivity

P	Total air pressure	hPa
T	Temperature	K
e_v	Partial pressure of water vapor	hPa

- Neutral atmosphere (non-dispersive):

$$n - 1 = 10^{-6} N, \qquad N = N_{\text{dry}} + N_{\text{wet}} = k_1 \frac{P}{T} + k_2' \frac{e_v}{T} + k_3 \frac{e_v}{T^2}.$$

Constants (for P, e_v in hPa):

$$k_1 = 77.6\ \text{K hPa}^{-1}, \quad k_2' = 70.4\ \text{K hPa}^{-1}, \quad k_3 = 3.739 \times 10^5\ \text{K}^2\ \text{hPa}^{-1}.$$

Troposphere: Zenith Delays

- Zenith delays: $ZHD = 10^{-6} \int N_{\text{dry}}\, dz, \qquad ZWD = 10^{-6} \int N_{\text{wet}}\, dz.$

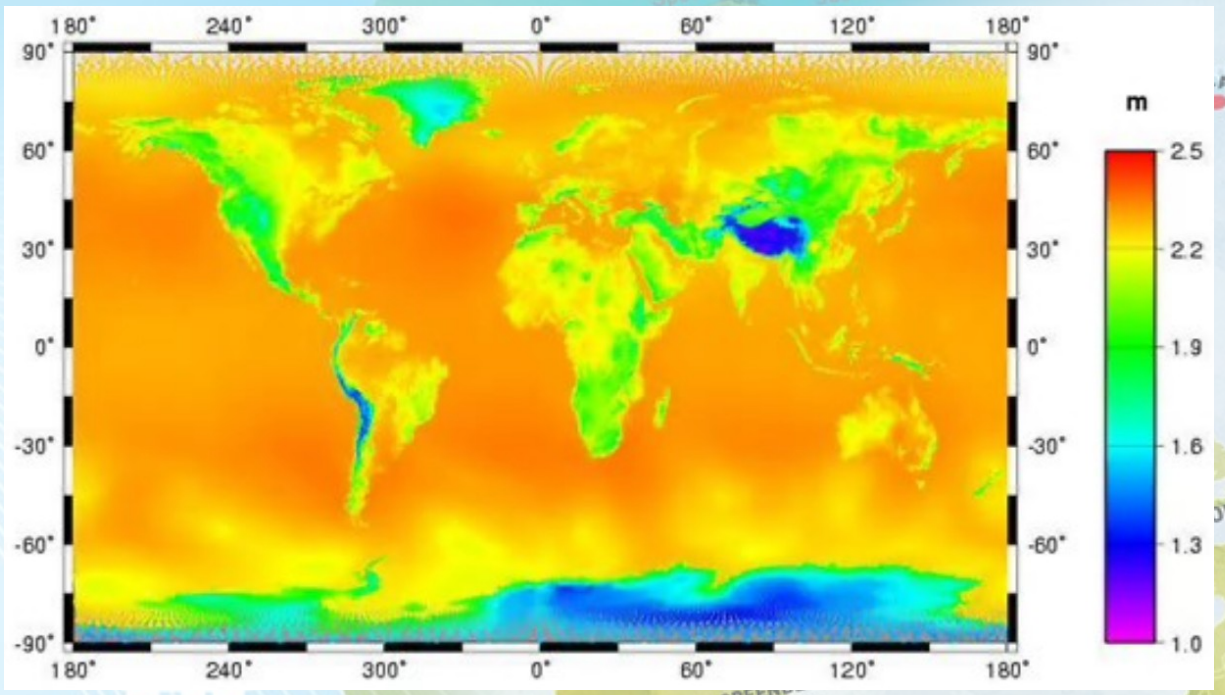
Zenith Hydrostatic Delay Zenith Wet Delay

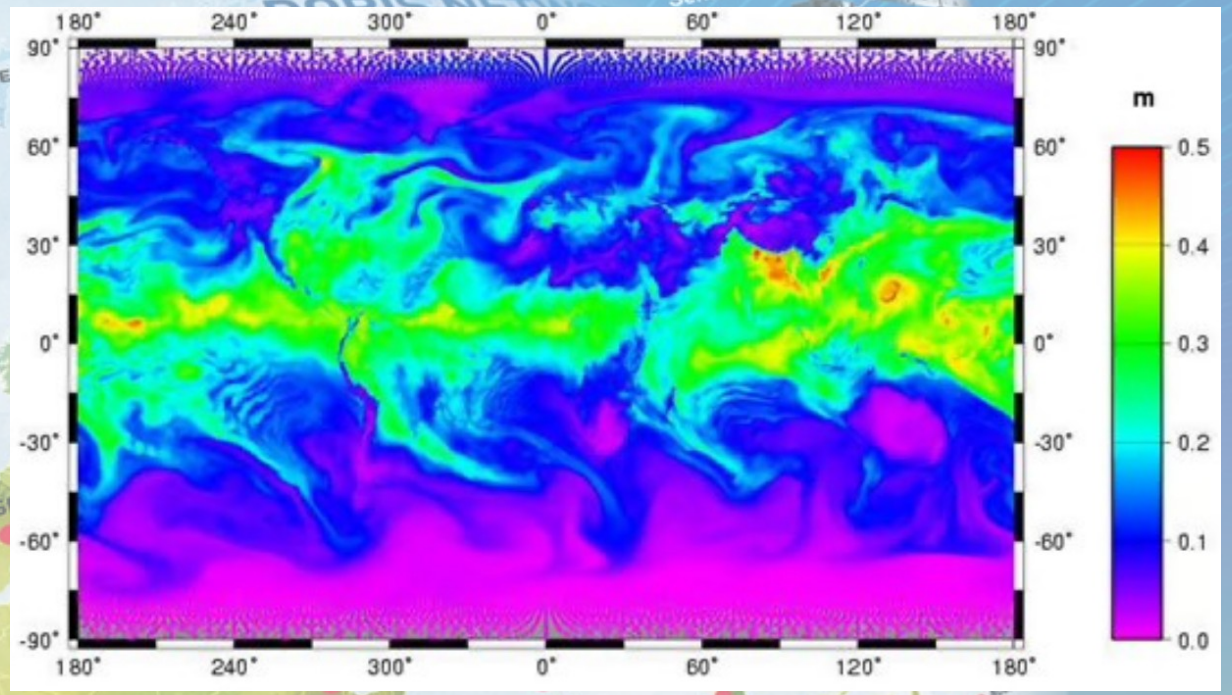
- Operational (Saastamoinen 1972) under the **hydrostatic equilibrium assumption** :

$$ZHD \approx 0.002277 \, \frac{P_0}{1 - 0.00266 \cos 2\varphi - 0.00028\, h} \quad [\text{m}]$$

- **ZWD** can be obtained from meteorological or NWM data, but **must be estimated as a parameter during geodetic data processing**.

Troposphere: Zenith Delays ZHD & ZWD

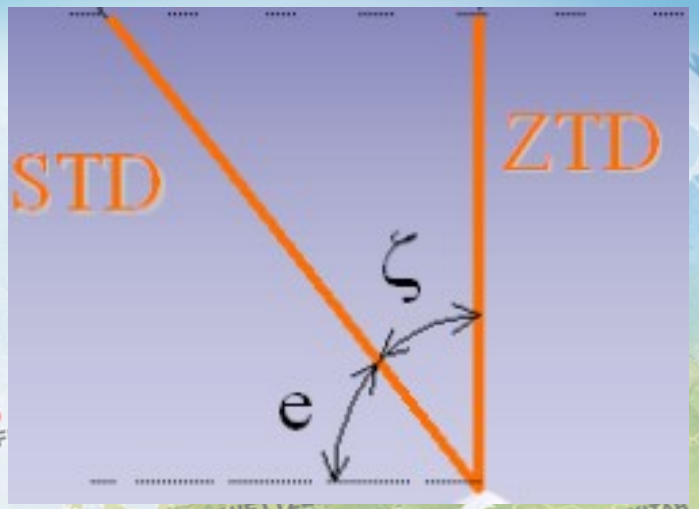




From ECMWF [Nahmani, 2005]

Tropospheric Mapping Function Geometry

$$m(e) \equiv \frac{STD}{ZTD} = \frac{1}{\sin(e)}$$



$$SHD = \frac{ZHD}{\sin\epsilon}, \qquad SWD = \frac{ZWD}{\sin\epsilon}, \qquad L_{\text{trop}}(\epsilon) = SHD + SWD$$

Right-triangle geometry shows how elevation controls the slant delay.

Refraction curvature is neglected in this simple model; real mapping functions refine this dependence. (VMF1 – Böhm & Schuh., 2004 ; GMF – Böhm et al., 2006 ; GPT2 – Lagler et al., 2013 ; VMF3/GPT3 – Landskron & Böhm, 2018)

Troposphere: Line-of-Sight Mapping (Operational)

For elevation angle ϵ:

$$\boxed{L_{\text{trop}}(\epsilon) = ZHD\, m_h(\epsilon) + ZWD\, m_w(\epsilon)}.$$

10-s variation (for Doppler):

$$\boxed{\dot{L}_{\text{trop}} \approx ZHD\, \frac{\Delta m_h}{T_{\text{int}}} + ZWD\, \frac{\Delta m_w}{T_{\text{int}}}}, \qquad \Delta m_i = m_i(\epsilon_{t_0+T_{\text{int}}}) - m_i(\epsilon_{t_0}).$$

During $T_{\text{int}} = 10$ s, ZHD and ZWD are almost constant;

the variation stems from **geometry** through $m(\epsilon)$.

Ionospheric Delay

$$L_{\text{ion}}(f) = \frac{40.3}{f^2} TEC$$

- $TEC = \int N_e \, ds$: Total Electron Content (electrons · m^{-2})
- N_e : electron density (electrons · m^{-3})
- Delay is **dispersive** and inversely proportional to f^2.
- Phase and group delays have opposite signs.

$$L_{\text{ion}}(f_2) / L_{\text{ion}}(f_1) = \left(\frac{f_1}{f_2} \right)^2$$

For DORIS: $f_1 = 2.03625$ GHz, $f_2 = 401.25$ MHz.

Iono-Free Combination (with proof) 1/2

- Model per frequency (group delay / path length):

$$L_i = L_0 + \underbrace{\frac{40.3}{f_i^2} TEC}_{\text{1st-order ionosphere}}, \qquad L_0 \equiv \rho + L_{\text{trop}} + L_{\text{rel}} + L_{\text{inst}}.$$

- Define the iono-free combination:

$$L_{\text{IF}} \overset{\text{def}}{=} \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2}.$$

- We compute :

$$\begin{aligned} f_1^2 L_1 - f_2^2 L_2 &= f_1^2 \left(L_0 + \frac{40.3}{f_1^2} TEC \right) - f_2^2 \left(L_0 + \frac{40.3}{f_2^2} TEC \right) \\ &= (f_1^2 - f_2^2) L_0 + 40.3\, TEC\, (1 - 1) \\ &= (f_1^2 - f_2^2) L_0. \end{aligned}$$

Iono-Free Combination (with proof) 2/2

Divide by $f_1^2 - f_2^2$:

$$\boxed{L_{\mathrm{IF}} = L_0 = \rho + L_{\mathrm{trop}} + L_{\mathrm{rel}} + L_{\mathrm{inst}}}$$

→ the $1/f^2$ ionospheric term **cancels exactly**.

Doppler (rate) form uses the same linearity:

$$\boxed{\frac{f_{r,\mathrm{IF}} - f_{\mathrm{IF}}}{f_{\mathrm{IF}}} = -\frac{1}{c}\left(\dot{\rho} + \dot{L}_{\mathrm{trop}} + \dot{L}_{\mathrm{rel}} + \dot{L}_{\mathrm{inst}}\right)}$$

(first-order ionosphere removed).

Key points

- The DORIS Doppler observable measures $\dot{L}$, not L.
- **Troposphere**: non-dispersive but variable with elevation and humidity.
- **Ionosphere**: dispersive $(1/f^2)$ but corrected through dual-frequency combination.
- Final model:

$$\frac{f_r - f_e}{f_e} = -\frac{1}{c}\left(\dot{\rho} + \dot{L}_{\text{trop}} + \dot{L}_{\text{rel}} + \dot{L}_{\text{inst}}\right)$$