THE DETERMINATION OF THE ORBITAL ELEMENTS USING GPS

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1 INTRODUCTION

The satellite MIMOSA (Micro Measurements Of Satellite Acceleration) has been developed in the Astronomical Institute of the Academy of Sciences of the Czech Republic during recent years for better knowledge of non-gravitational disturbances: atmospheric constellation and radiation. The project is described in Sehnal et al. (1999) in detail. The position and velocity components of the satellite have to be determined by GPS (Global Positioning System) tracking instrument.

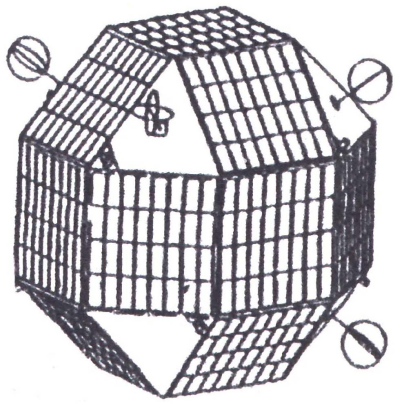

Figure 1: The MIMOSA satellite.

The MIMOSA project consists of the construction of a small satellite (polyhedron with a circumscribed diameter 60 cm), see Fig. 1, with the micro-accelerometer in the center of gravity. The principal tests of the micro-accelerometer have been done on board the Russian satellite Resource T-1 and on board the American Space Shuttle Atlantis in 1996. These experiments showed the expected properties of the device and its measuring capabilities. The sensitivity of the accelerometer is 10^{-11} ms^{-2} and the measuring range is between ± 0.4 ms^{-2}. The launch is expected

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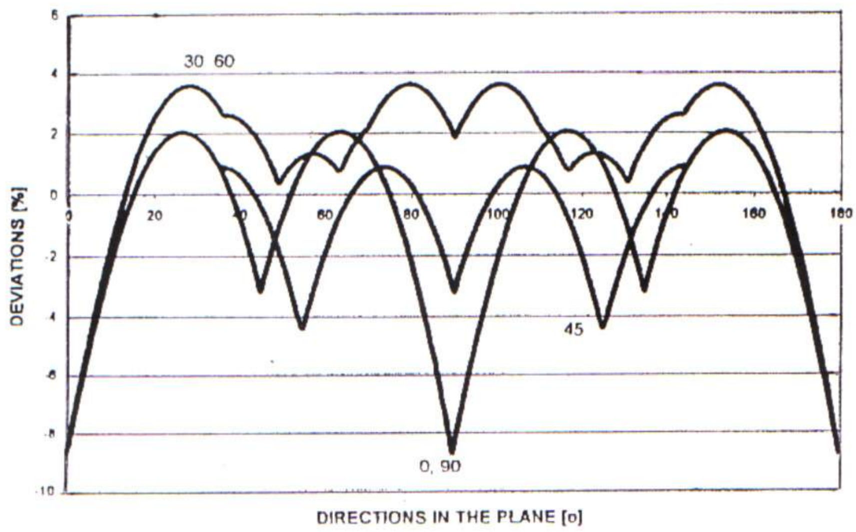

Figure 2. The deviations of the MIMOSA satellite's cross-sectional area as a function of orientation. The numbers 0, 30,....90 are the inclinations of the satellite's orbit towards the equatorial plane of the satellite

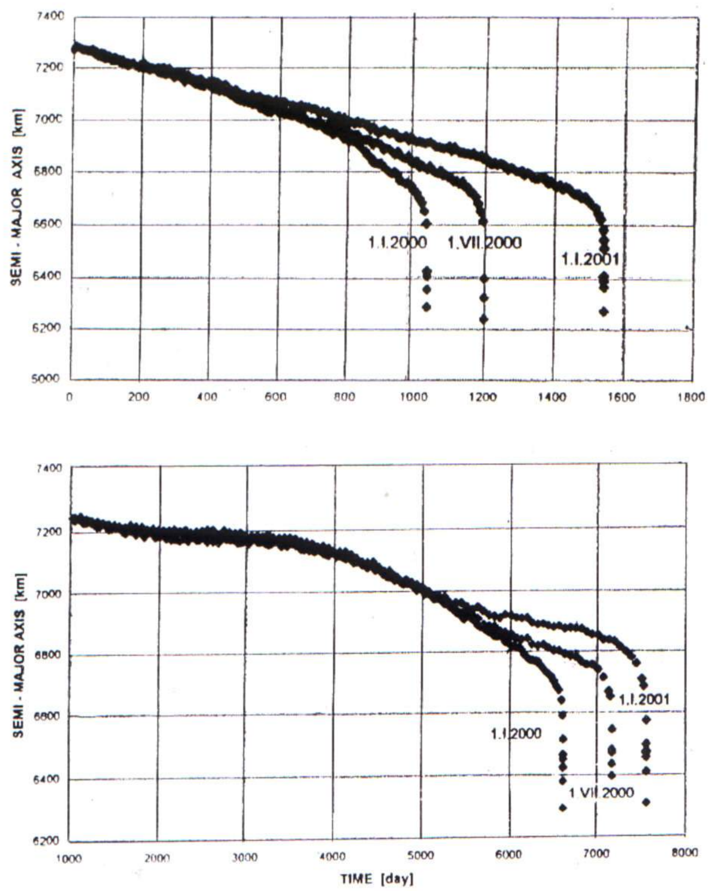

Figure 3: Semi-major axis as a function of time for 1^{st} (on the top) and 2^{nd} (on the bottom) variant of the orbital elements for the three launch dates for the MIMOSA satellite

In parallel with the technical developments, theoretical studies have been carried out concerning the atmospheric drag effects, the satellite's instantaneous area in Fig. 2, the atmospheric density, the rotation velocity of the atmosphere, the atmospheric streams, the lifetime of MIMOSA in Fig. 3, see Kabeláč and Sehnal (2003), the albedo, the infrared and direct solar radiation, see Sehnal et al. (1999). The input orbital elements were: $\Omega = 0°$, $I = 0°$, $M_0 = 0°$ and $H_{\rm peri} = 300$ km, $H_{\rm apog} = 1500$ km for the 1st variant and $H_{\rm peri} = 400$ km, $H_{\rm apog} = 1400$ km for the 2nd variant.

For all of these purposes, knowledge of the orbital elements is necessary.

Table 1: Time of the start, transit and its corrections.

I	J	Start [h]	Transit [ns]	Refr.	Cent.	Delay[ps]
1	1	2.88090789180556	53247355.344	35738	1650	121696
1	2	2.88154700047222	53220950.251	35666	1650	121718
1	3	2.88293589297222	53163727.841	35523	1650	121766
1	4	2.88413011155556	53114702.652	35378	1650	121808
1	5	2.88526922625000	53068092.415	35253	1650	121847
1	6	2.89129679622222	52823961.357	34609	1650	122059
1	7	2.89671366347222	52608206.367	34055	1650	122253
2	8	11.9970139858333	39923201.300	16043	1601	-218495
2	9	11.9970834202778	39923054.500	16043	1601	-218495
2	10	11.9974306427778	39922329.300	16042	1601	-218495
2	11	11.9979167541667	39921352.900	16041	1601	-218495
2	12	11.9979861975000	39921216.200	16041	1601	-218495
2	13	11.9981945322222	39920814.100	16041	1601	-218495
2	14	11.9986806302778	39919907.000	16040	1601	-218495
3	15	14.0312293970278	44495418.806	17806	1674	18184
3	16	14.0330628535833	44456871.776	17768	1674	18184
3	17	14.0352294693889	44411988.986	17724	1674	18184
3	18	14.0353960783889	44408567.656	17720	1674	18184
3	19	14.0357850108889	44400597.716	17713	1674	18184
3	20	14.0363406202778	44389252.756	17702	1674	18184
3	21	14.0370072033333	44375705.216	17688	1674	18184
4	22	22.8723340871389	52682900.889	30026	1674	84151
4	23	22.8726118649722	52698877.079	30055	1674	84151
4	24	22.8726674205278	52702072.919	30061	1674	84151
4	25	22.8729451984167	52718054.639	30093	1674	84151
4	26	22.8732785316389	52737238.549	30131	1674	84151
4	27	22.8733896427500	52743634.559	30144	1674	84151
4	28	22.8735563094722	52753230.109	30163	1674	84151

2 THE FUNDAMENTAL IDEAS OF THE THEORY

We used the equations

$$X = FX_0 + G\dot{X}_0 + S_X, \quad Y = FY_0 + G\dot{Y}_0 + S_Y, \quad Z = FZ_0 + G\dot{Z}_0 + S_Z \qquad (1)$$
$$\dot{X} = \dot{F}X_0 + \dot{G}\dot{X}_0 + S_{\dot{X}}, \quad \dot{Y} = \dot{F}Y_0 + \dot{G}\dot{Y}_0 + S_{\dot{Y}}, \quad \dot{Z} = \dot{F}Z_0 + \dot{G}\dot{Z}_0 + S_{\dot{Z}}$$

for determination of the satellite's position and velocity at the arbitrary time. The equations (1) without $\mathrm{S}_{\rm index}$ are valid only for Keplerian motion. The $\mathrm{S}_{\rm index}$ are the

Table 2: Convergence of the single orbital elements

X_0 [m]	Y_0 [m]	Z_0 [m]
-11218448.7053	-1583779.2744	4280465.2898
-11218460.4471	-1583567.9789	4280462.6717
-11218463.2119	-1583559.9940	4280460.8343
-11218463.1466	-1583560.1368	4280460.8654
-11218463.1509	-1583560.1227	4280460.8631
-11218463.1508	-1583560.1230	4280460.8631
.0001	.0003	.0000
$\dot{X}_0$ [m/s]	$\dot{Y}_0$ [m/s]	$\dot{Z}_0$ [m/s]
1984.99847482	-3484.90126424	4119.41360278
1984.94609415	-3484.95083402	4119.40736071
1984.94304201	-3484.94915673	4119.40962101
1984.94311133	-3484.94916773	4119.40959861
1984.94310601	-3484.94916560	4119.40960201
1984.94310613	-3484.94916562	4119.40960196
.0000001	.00000002	.00000005

disturbances. We included in these disturbances the influences of the Earth, Moon and Sun, the radiations of the Sun and Earth and the influence of the atmosphere. The symbol $\dot{A}$ denotes the derivation in the time of A.

First of all we took an interest in the determination of the orbital elements. We used the method of numerical integration by Everhart (1974), five kinds of perturbations and the least square method for the adjustment.

3 ORBIT DETERMINATION

3.1 SATELLITE LASER RANGING

The measured value is the distance between the station and the satellite LAGEOS 2 and between LAGEOS 2 and the station. For verification of the whole numerical process we used measurements of the Canberra, Chanchung, Arequipa and Quincy stations on September 7, 1995, see Tab. 1. Table 2 shows the convergence of the orbital elements after a single adjustment. For more details, see in Sborník (2002).

3.2 USING THE GPS METHOD

This method is described in Kabeláč (2003). We used a fictive case because we were not able to get real observations. Of course, we used also the Eq. (1), the same method of numerical integration, the same perturbations and the least square method. The equations of the errors for the unconditioned adjustment were used. In the rows of the Tab. 3 are: 1. correct values, 2. the introduced errors, 3. the input incorrect values, 4. the corrections from the 1st adjustment, 5. the result from the 1st adjustment, 6. the error from 2nd adjustment, 7. the corrections from the 2nd adjustment, 8. the result after the 2nd adjustment and 9. the (final) error after the 2nd adjustment.

Table 3: The single orbital elements adjustments.

	X_0 [m]	Y_0 [m]	Z_0 [m]	$\dot{X}_0$ [m/s]	$\dot{Y}_0$ [m/s]	$\dot{Z}_0$ [m/s]
1	-14889085	-3660180	21848509	496.538	-3810.433	-305.561
2	100	-100	100	0.15	-0.10	0.10
3	-14888985	-3660280	21848609	496.688	-3810.533	-305.461
4	-100.928	99.697	-99.878	-0.15	0.1	-0.10
5	-14889085.928	-3660180.303	21848509.122	496.538	-3810.433	-305.561
6	-0.928	-0.303	0.122	0	0	0
7	0.93	0.303	-0.119	3E-8	1E-7	1E-7
8	-14889085.002	-3660180	21848509.003	496.538	-3810.433	-305.561
9	-2mm	0mm	3mm	0mm/s	0mm/s	0mm/s

4 CONCLUSIONS

It is sure that the verification of the numerical process in the last case is imperfect. Use of the values measured in real time is unavoidable. This is the first point, and secondly: connection with the other disturbances is recommended. In the future: we recommend transforming the presented computation programs (for determination of the orbital elements) into a new program to measure components of velocity via DORIS. This task is set up in the frame of the tasks of the Center for Earth's Dynamics Research, Kostelecký (2003).

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