Statistical studies of the DORIS position time series

Spectral characteristics and comparison between three Analysis Centres solutions

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Contents

- The 3 AC solutions : IGN-JPL, INASAN, LCA
- Comparison of these solutions : annual terms and temporal correlations
- Noise analysis :
  - Method : Principal Component Analysis applied in the time domain and Allan variance
  - Results : DORIS type and level of noise, relationship between the level of noise and some technical and geographical features
  - Stability index per station

⇨ *Possible consequences on a combined solution*

Time series

Analysis Centres	Data span	Stations #	Software
IGN-JPL (ignwd05)	93.0-05.2	119	GIPSY/OASIS
INASAN (inawd03)	92.8-04.4	111	GIPSY/OASIS
LEGOS-CLS (lcawd12)	93.0-05.0	114	GINS/DYNAMO

- Residuals relative to a linear motion model for the station motion

Part I : comparison of the three sets of solutions

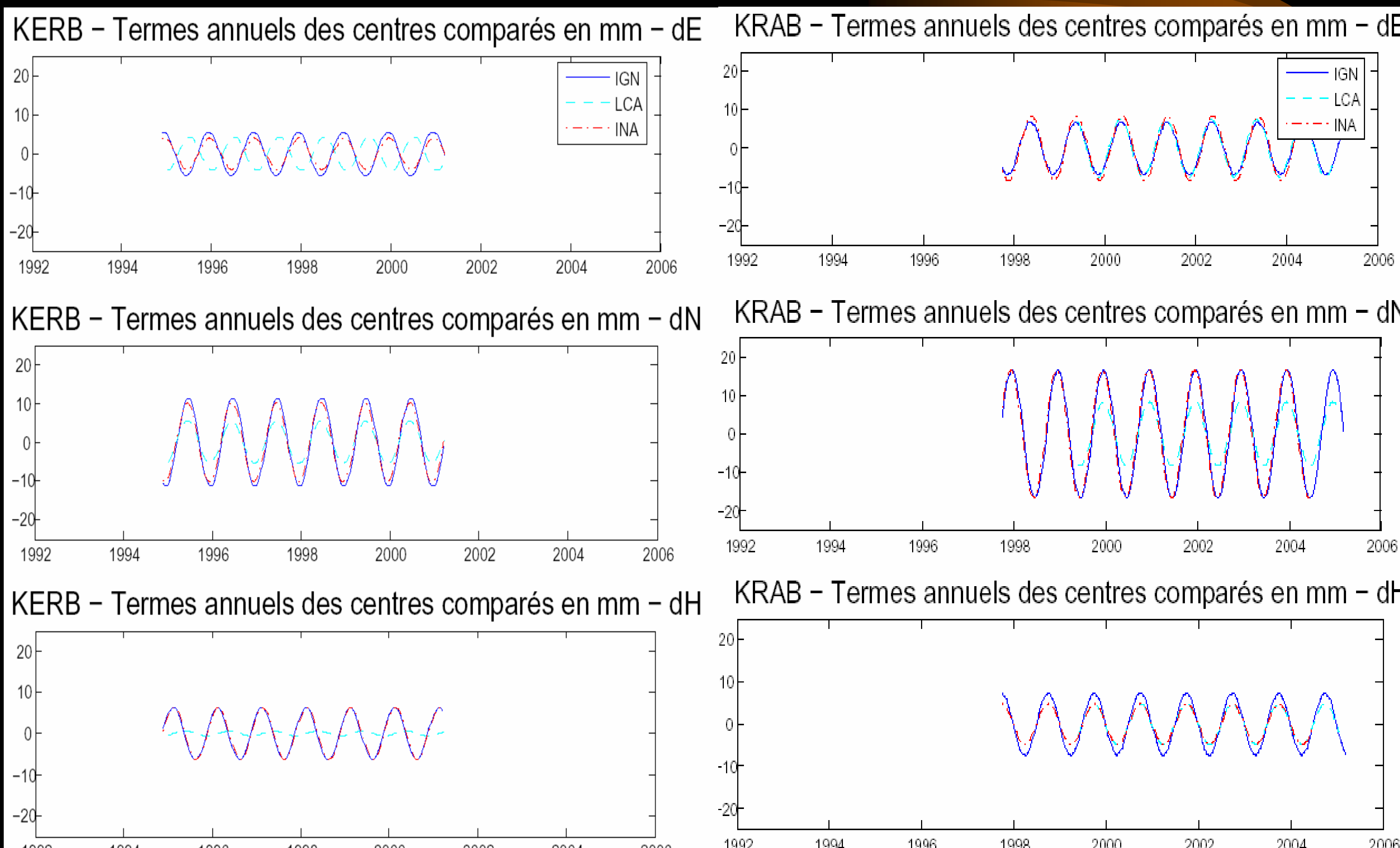
Used tools :

- ✓Annual terms determination (Least Square)
- ✓Temporal correlation

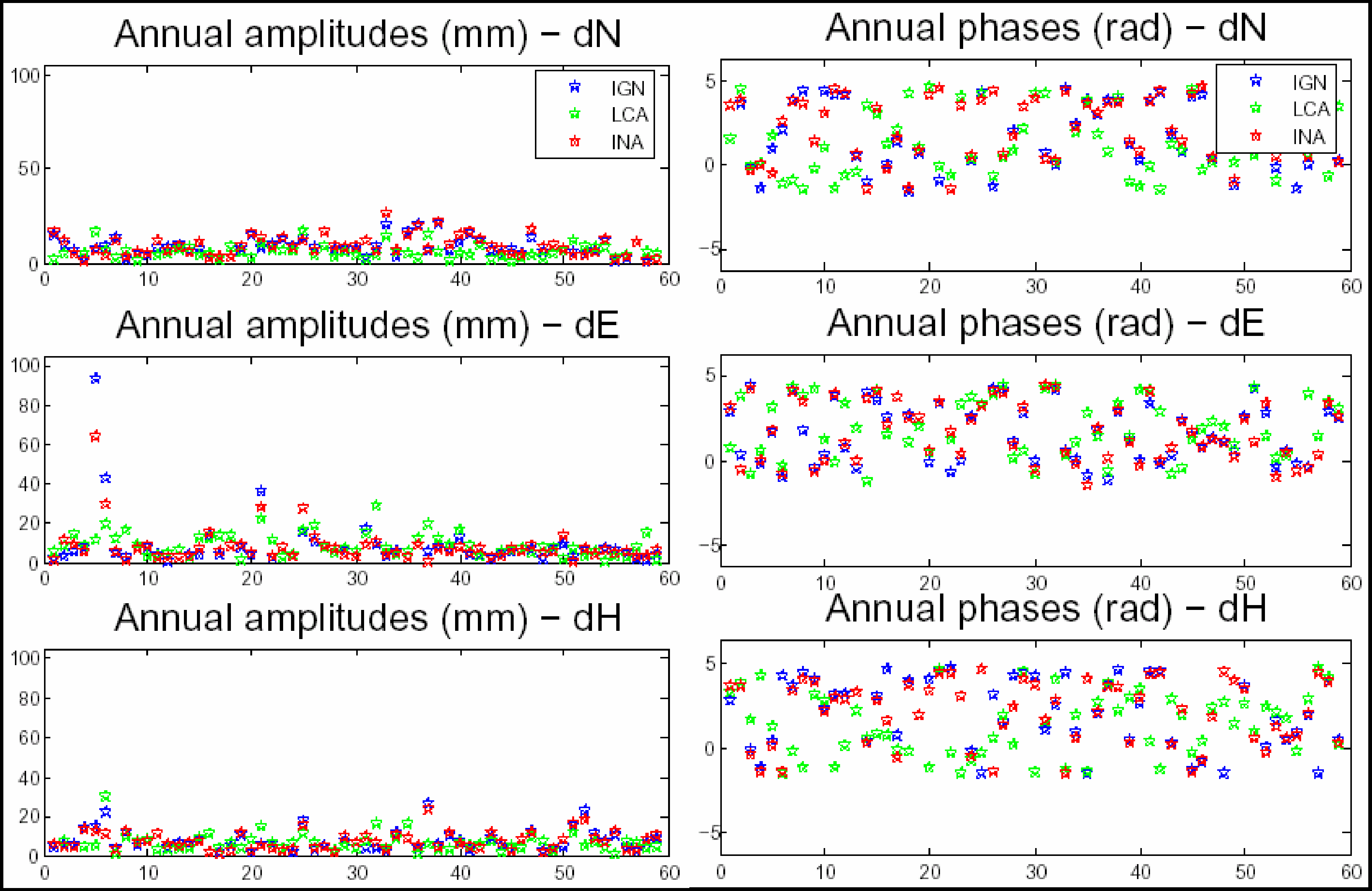
- ✓107 common stations, 59 longer than 3 years

Engineer diploma, W. Zerhouni

Annual terms determination.
Two examples : KERB and KRAB



DORIS annual amplitudes and phases



Temporal correlations

- Temporal covariance :

$$cov(dE_{IGN}^{statj}, dE_{LCA}^{statj}) = \frac{1}{n}\sum_{i=1}^{n}(dE_{IGN}^{statj}(t_i) - \overline{dE_{IGN}^{statj}})(dE_{LCA}^{statj}(t_i) - \overline{dE_{LCA}^{statj}})$$

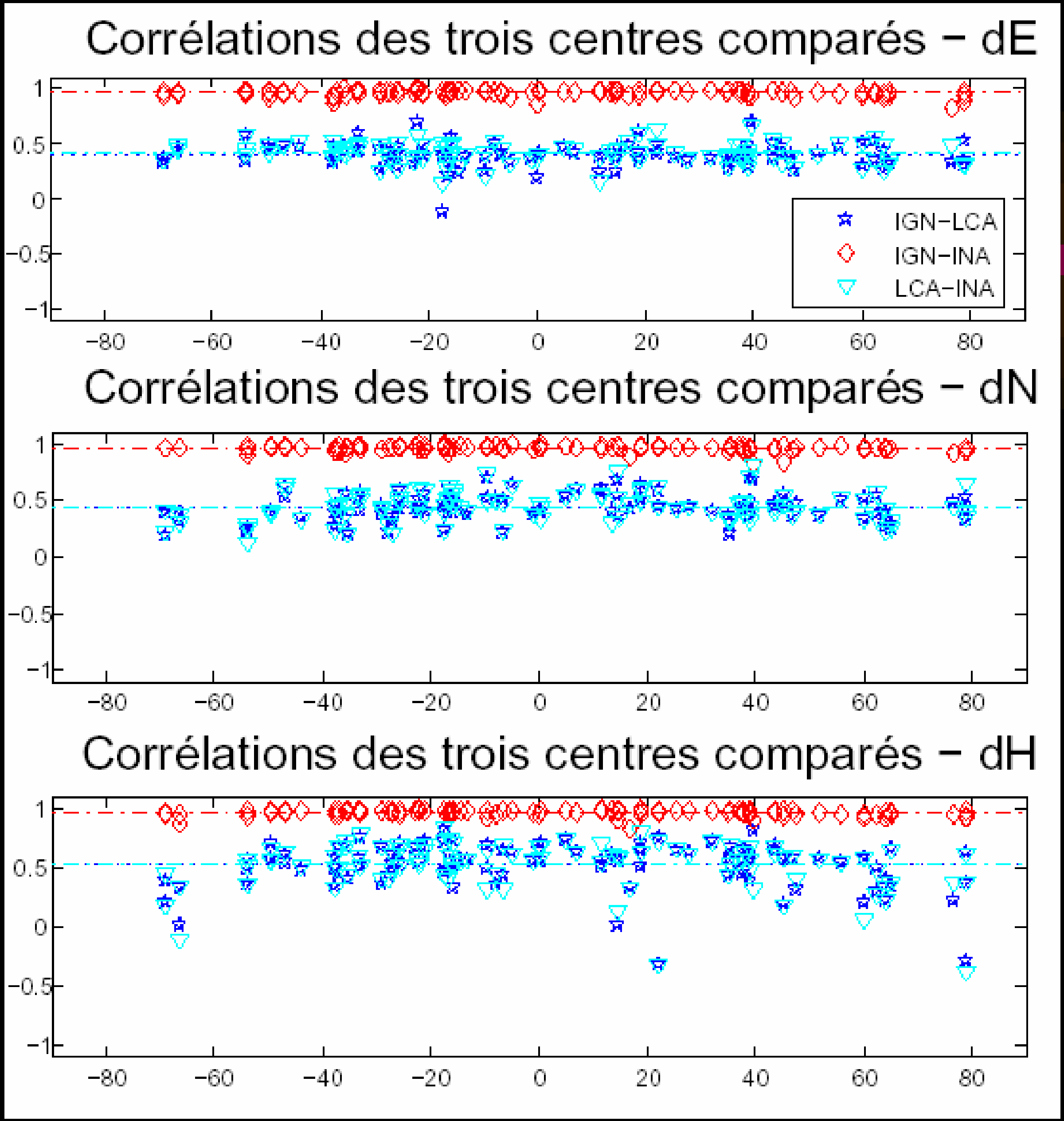
- Temporal correlation :

$$\rho = corr(dE_{IGN}^{statj}, dE_{LCA}^{statj}) = \frac{\mathrm{cov}(dE_{IGN}^{statj}, dE_{LCA}^{statj})}{\sqrt{\mathrm{cov}(dE_{IGN}^{statj}, dE_{IGN}^{statj})\,\mathrm{cov}(dE_{LCA}^{statj}, dE_{LCA}^{statj})}}$$

$\rho = 0$: the two time series are uncorrelated

$\rho = \pm 1$: the two time series are (anti)correlated

- We access to the agreement between solutions two by two and station by station.



- IGN-INA :
  High correlations from 0.70 to 0.99
- LCA-IGN/INA :
  - Fairly weak correlations from -0.40 to 0.75 with an average of 0.46
  - In vertical : discrepancies

Part II : statistical studies

Used tools :

- ✓Principal Component Analysis applied in the time domain
- ✓Allan variance

Criteria for statistics meaning :

- ✓C1 : longer than 3 years,< 30% missing weeks, data gap < 200 days
- ✓C2 : longer than 3 years, data gap < 400 days
- ✓C3 : other stations

	Total	**C1**	**C2**	**C3**
IGN-JPL	119	**55**	**9**	**55**
INASAN	111	**54**	**10**	**47**
LEGOS-CLS	114	**63**	**11**	**40**

PCA : Theory (1)

- Let $A(i,j)$ as :
  - $A(i,1)$ is the dN coordinate of the considered point at the date t_i ;
  - $A(i,2) = dE(t_i)$;
  - $A(i,3) = dU(t_i)$.
- Empirical variance-covariance matrix (in the time domain) :

$$COV_A(k,l) = \sum_{i=1}^{n} \frac{(A(i,k) - \overline{A}_k)(A(i,l) - \overline{A}_l)}{n}$$

$$cov(dE_{IGN}^{statj}, dN_{IGN}^{statj}) = \frac{1}{n} \sum_{i=1}^{n} (dE_{IGN}^{statj}(t_i) - \overline{dE_{IGN}^{statj}})(dN_{IGN}^{statj}(t_i) - \overline{dN_{IGN}^{statj}})$$

PCA : Theory (2)

- Eigenvectors and eigenvalues of the COV_A matrix
- Projection of each triplet $(dN(t_i), dE(t_i), dU(t_i))$ on the eigenspace generated by eigenvectors :

$$\begin{pmatrix} A(t_i) \\ B(t_i) \\ C(t_i) \end{pmatrix} = M^t \begin{pmatrix} dN(t_i) \\ dE(t_i) \\ dU(t_i) \end{pmatrix}$$

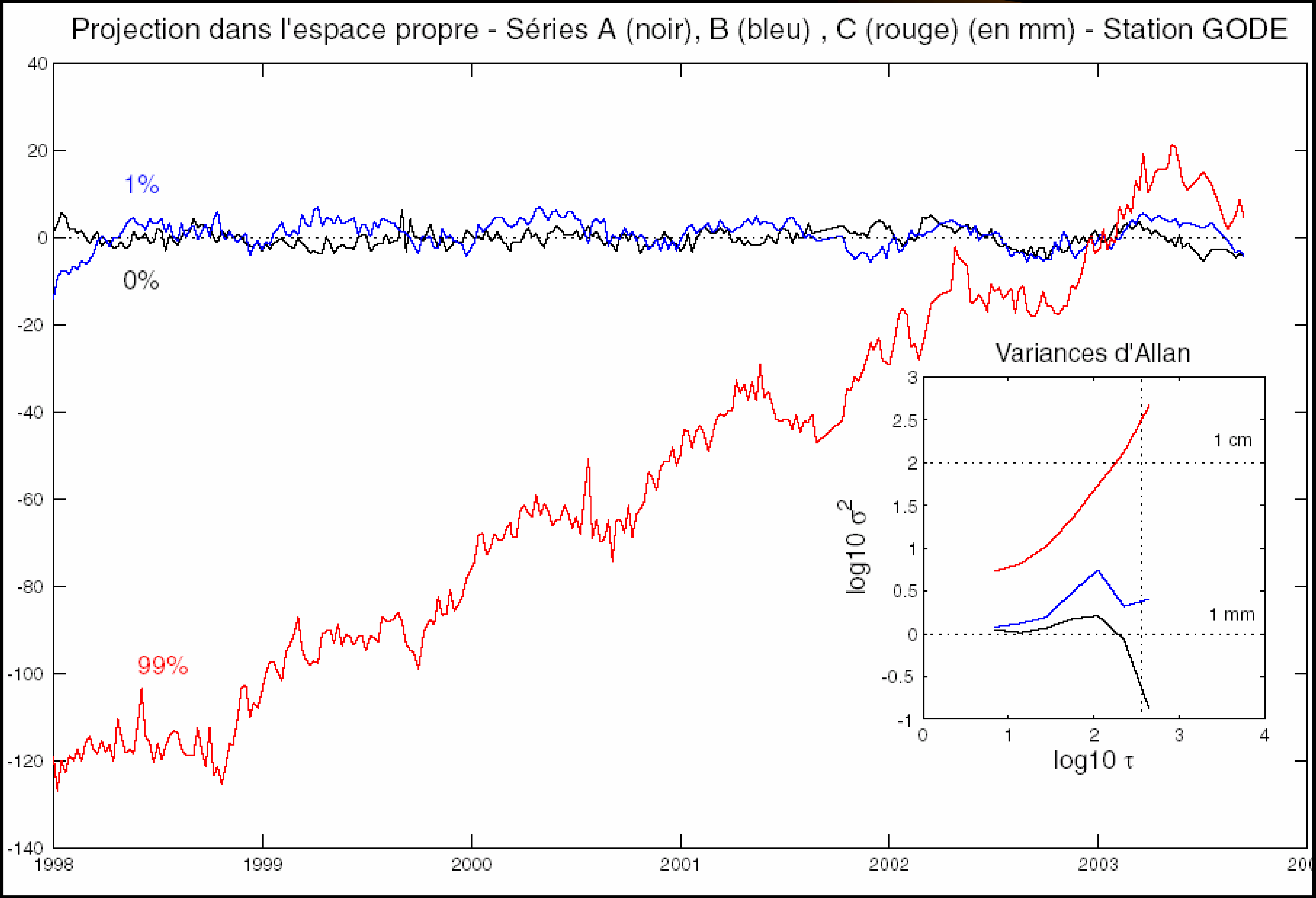
- We obtained three components for which the variance percentage is :

$$P_l = \frac{\lambda_l}{\sum_{m=1}^{3} \lambda_m}$$

Principal Component Analysis applied in the time domain

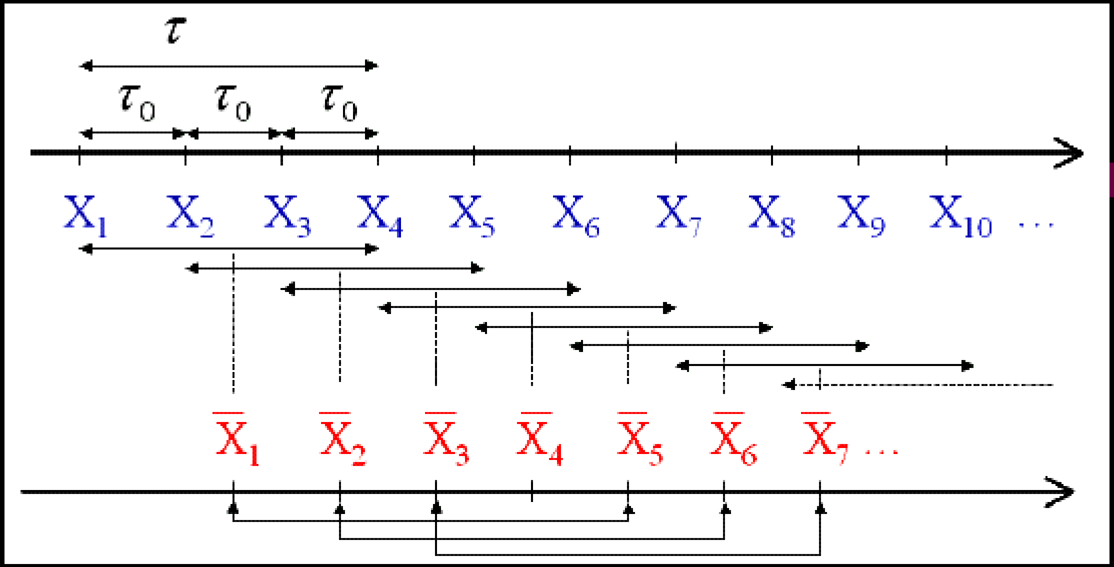
- Access to the principal component (PCT1) which represents the 3D most significant time behaviour of the series (maximum variance);
- Each obtained component is independent : no correlation.

The CODE « bug »



Projection dans l'espace propre - Séries A (noir), B (bleu) , C (rouge) (en mm) - Station GODE
1%
0%
99%
40
20
0
-20
-40
-60
-80
-100
-120
-140
1998
1999
2000
2001
2002
2003
Variances d'Allan
1 cm
1 mm
log10 σ^2
log10 τ
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
1
2
3
4

Allan variance



$(X_j)_{j \in I}$ are studied measurements

τ is the sampling time

- Allan variance :

$$\sigma_X^2(\tau) = \frac{1}{2} < (\bar{X}_{k+1} - \bar{X}_k)^2 >$$

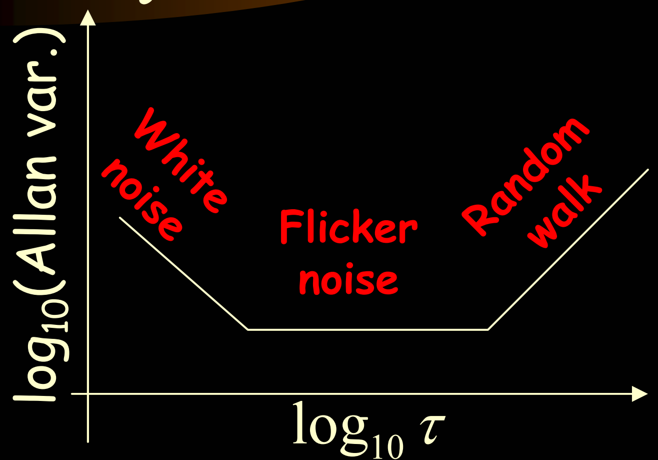
- Its graphical representation :

$$\log(\sigma^2(\tau)) = \mu \log(\tau),\ for\ \tau = \tau_0, 2\tau_0, 4\tau_0, \ldots$$

Allan variance and spectral density law

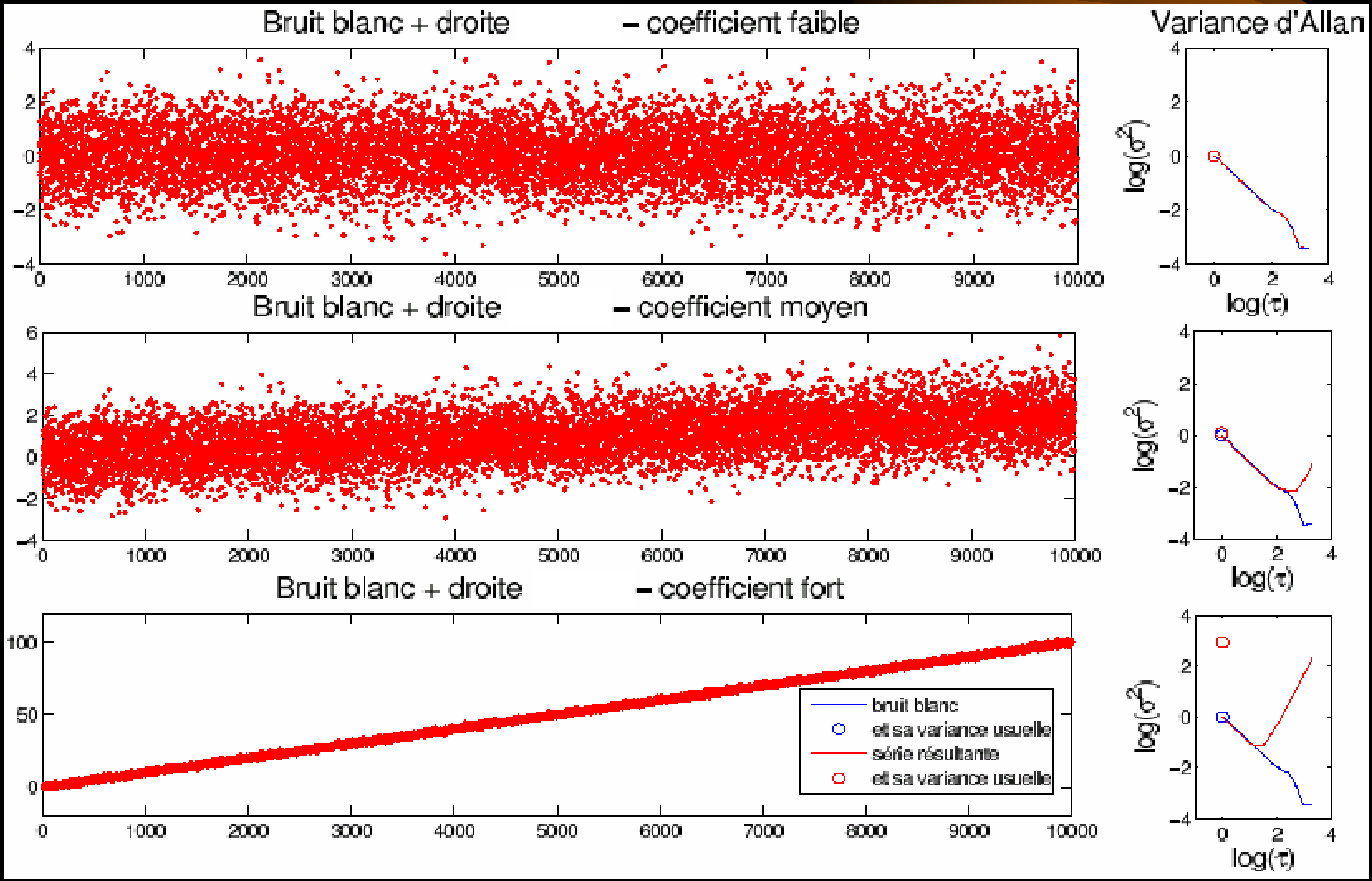
- Let $S_x(f) = h_\alpha f^\alpha$ the spectral density of the processus :

$$\sigma_x^2(\tau) = \frac{1}{card(I)\tau_0} \sum_{i \in I} S_x(f_i) \frac{2\sin^4(\pi\tau f_i)}{(\pi\tau f_i)^2}$$

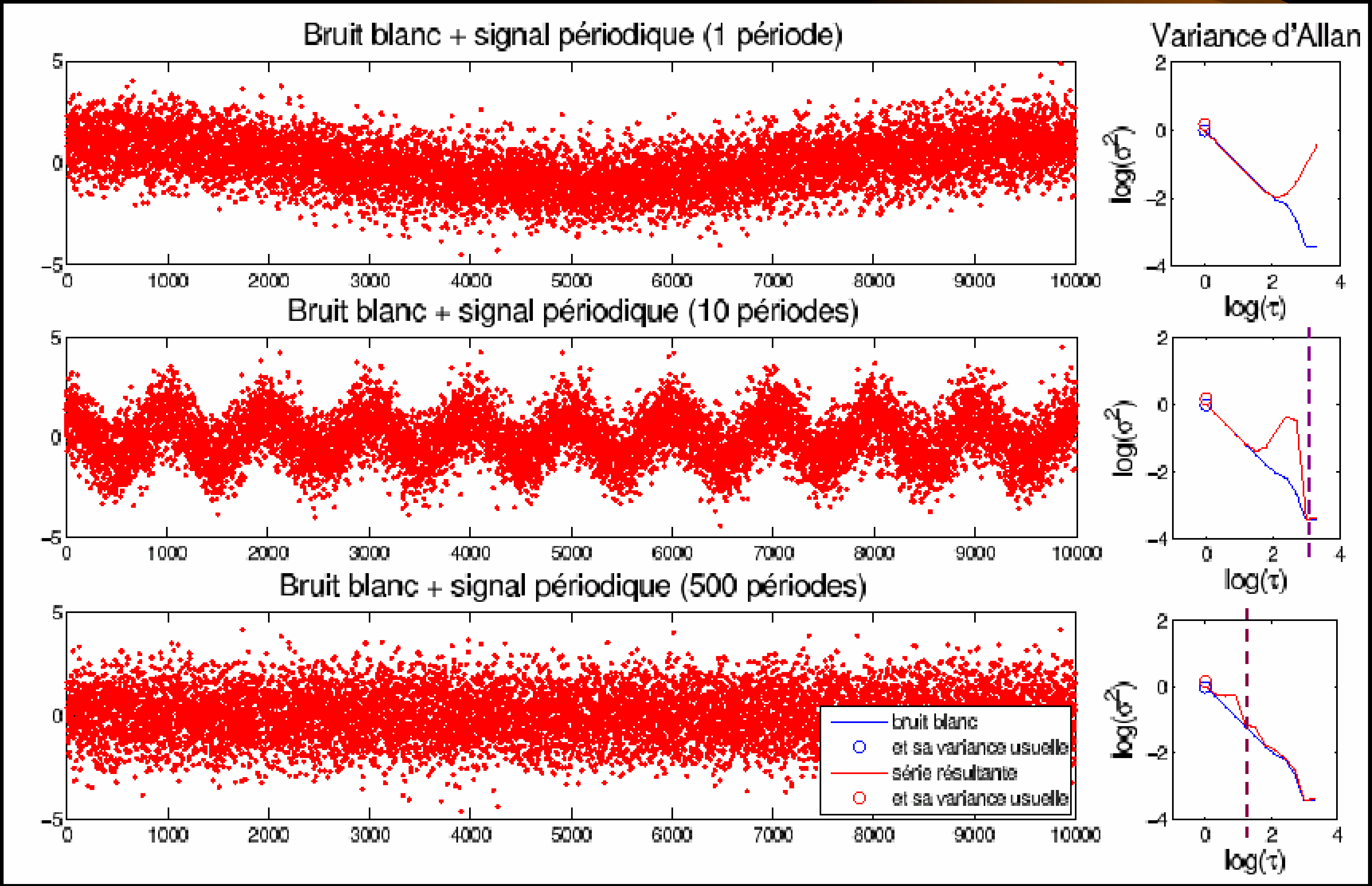


- Noise determination :
  - $\alpha = 0 \Leftrightarrow$ **white noise** $\Leftrightarrow \mu = -1$
  - $\alpha = -1 \Leftrightarrow$ **flicker noise** $\Leftrightarrow \mu = 0$
  - $\alpha = -2 \Leftrightarrow$ **random walk** $\Leftrightarrow \mu = 1$

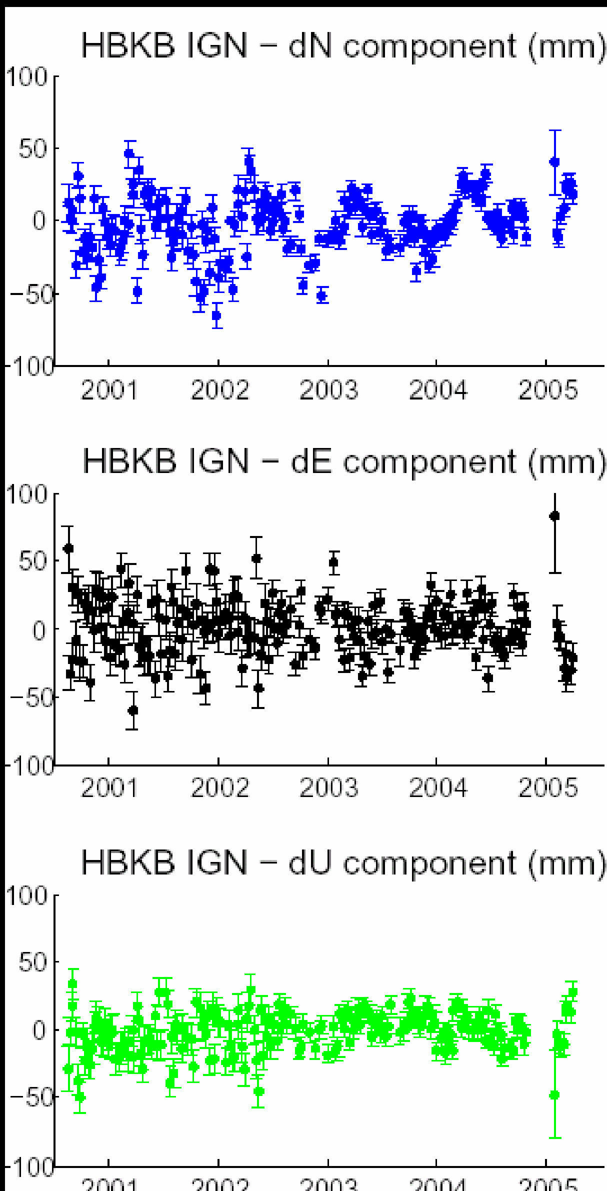
Sensitivity to a linear drift



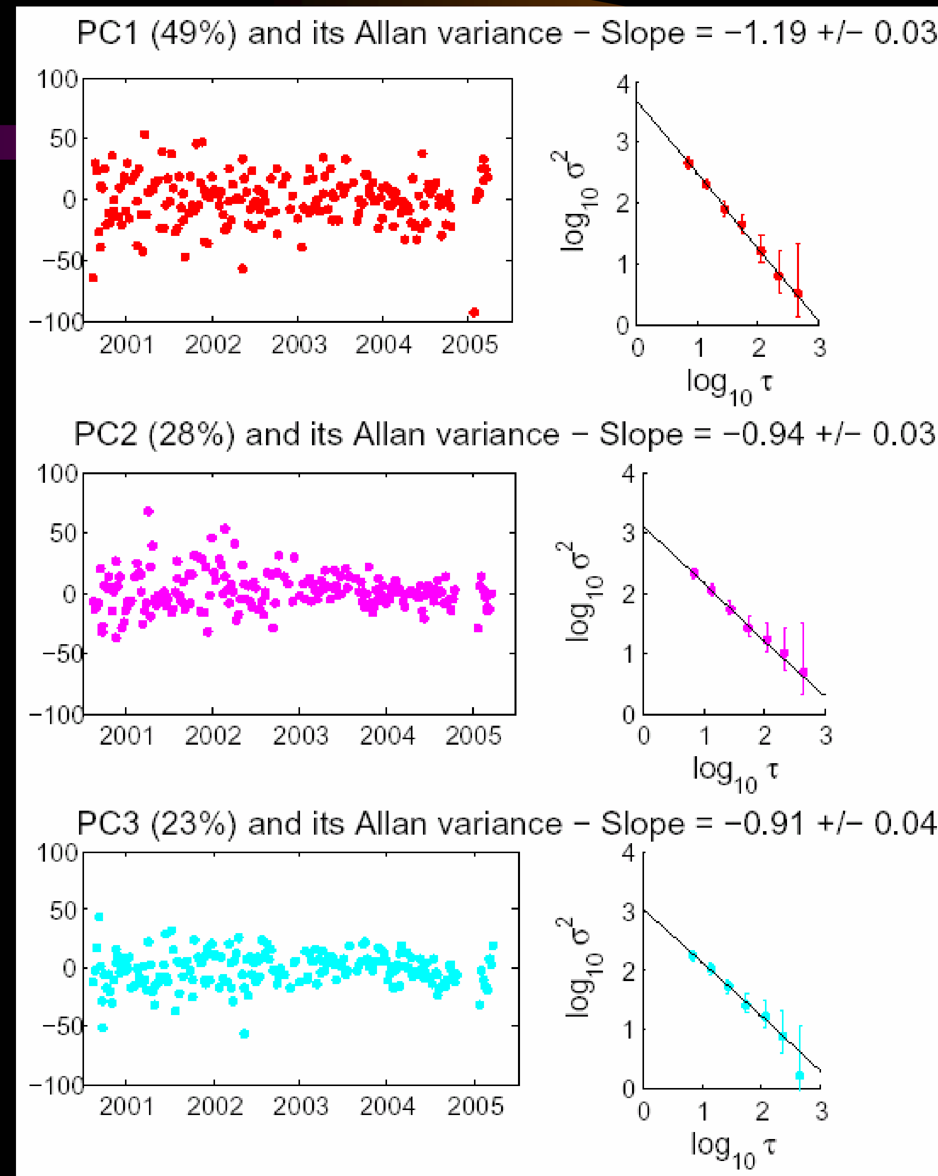
Sensitivity to a periodic signal



Application – HBKB Example (1)



ANALYSIS :
Cleaning + PCAt



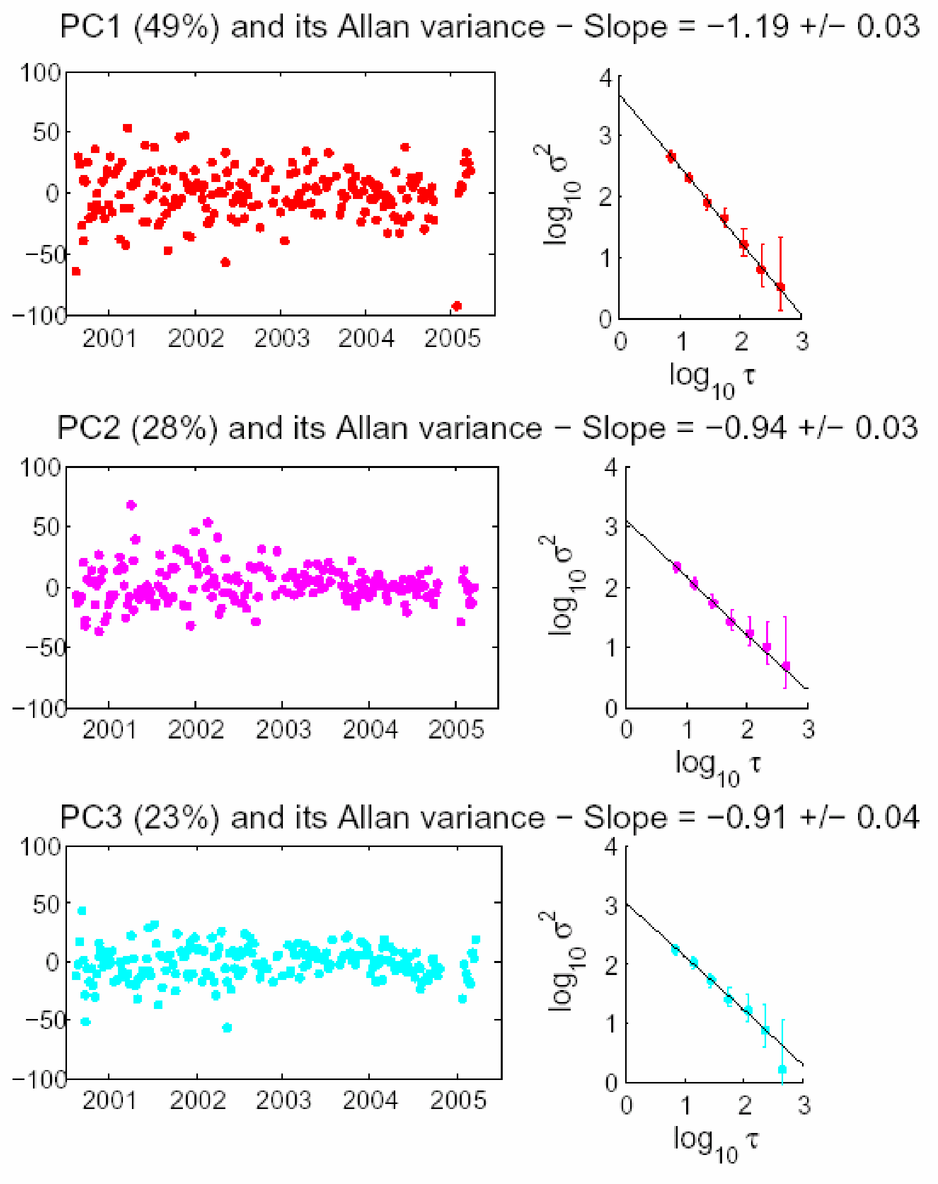
Application – HBKB Example (2)

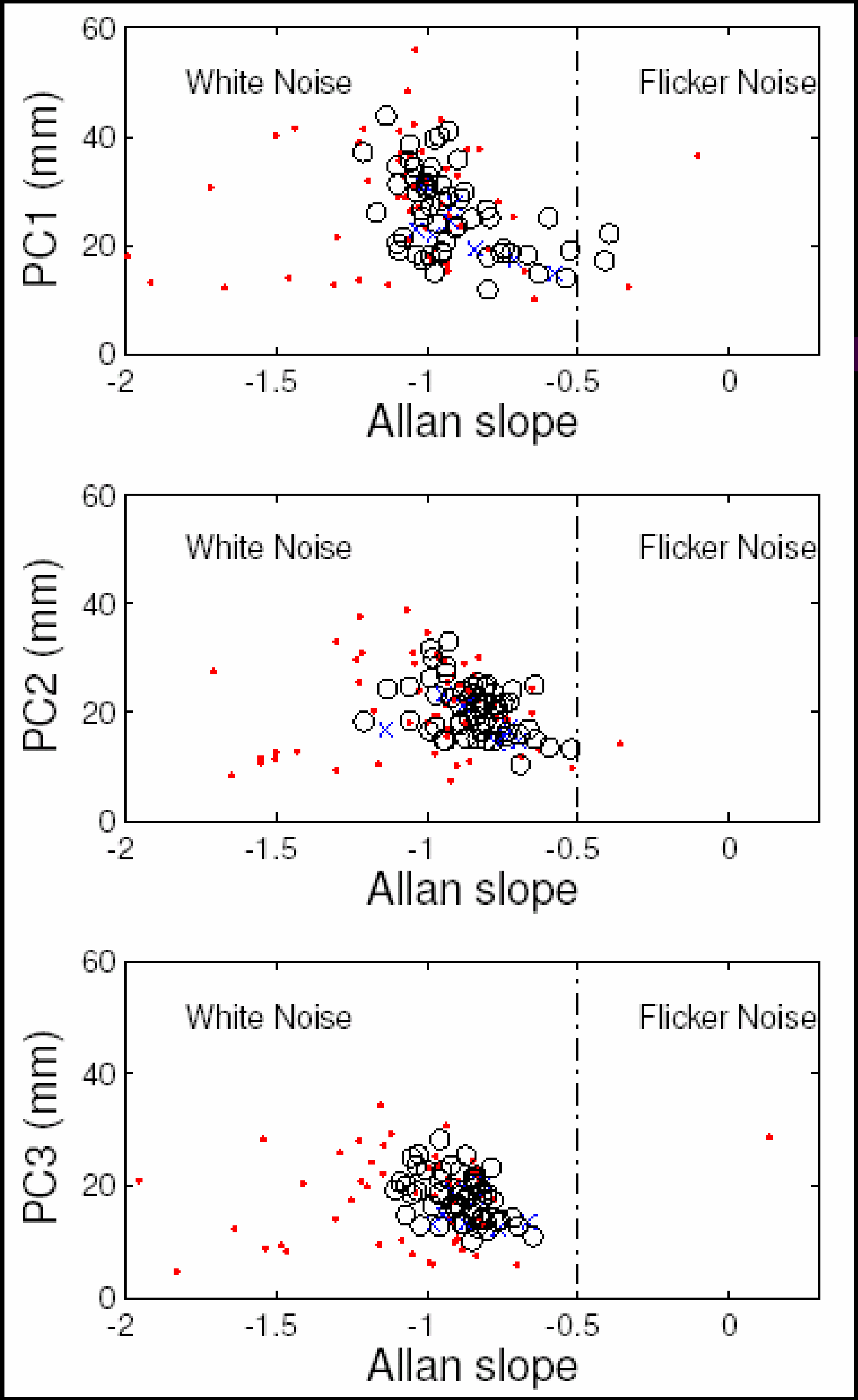
- Correlation matrix :

$$corr = \begin{pmatrix} 1 & 0.11 & -0.03 \\ 0.11 & 1 & -0.14 \\ -0.03 & -0.14 & 1 \end{pmatrix}$$

- Eigenvectors :

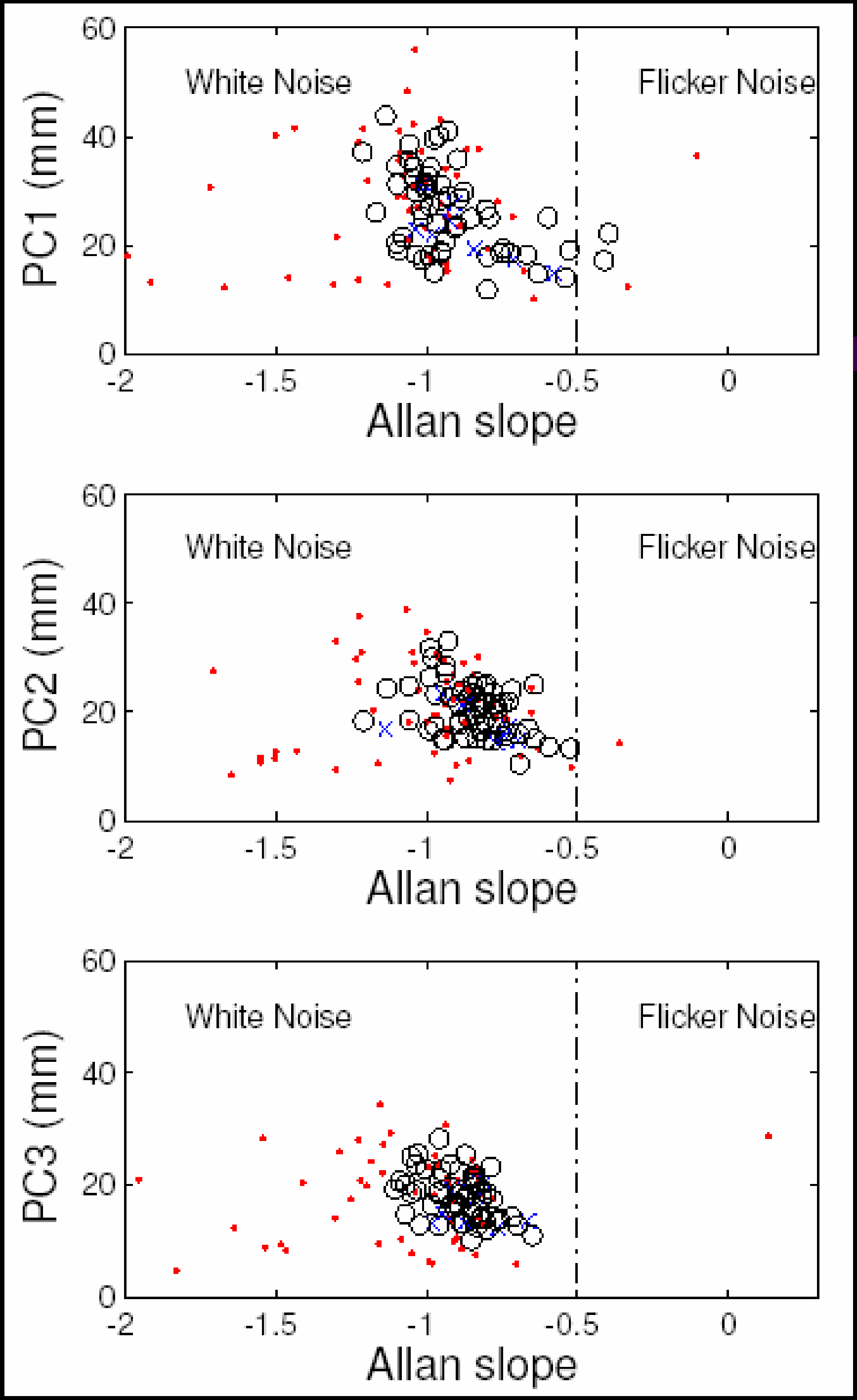
$$\begin{matrix} -0.04 & 0.18 & -0.98 & dN \\ 0.98 & 0.18 & -0.00 & dE \\ 0.18 & -0.96 & -0.19 & dU \\ v1 & v2 & v3 & \end{matrix}$$





Type and level of DORIS noise

- PC1 : 40 to 60% (explained variance), white noise at the level of 10 to 45 mm, except for two stations with 20 mm flicker noise : SYOB and OTTA;
- PC2 : 20 to 40%, 10 to 40 mm white noise;
- PC3 : 17 to 30%, 10 to 28 mm white noise.



Station's stability interpretation

- Stability index per station :

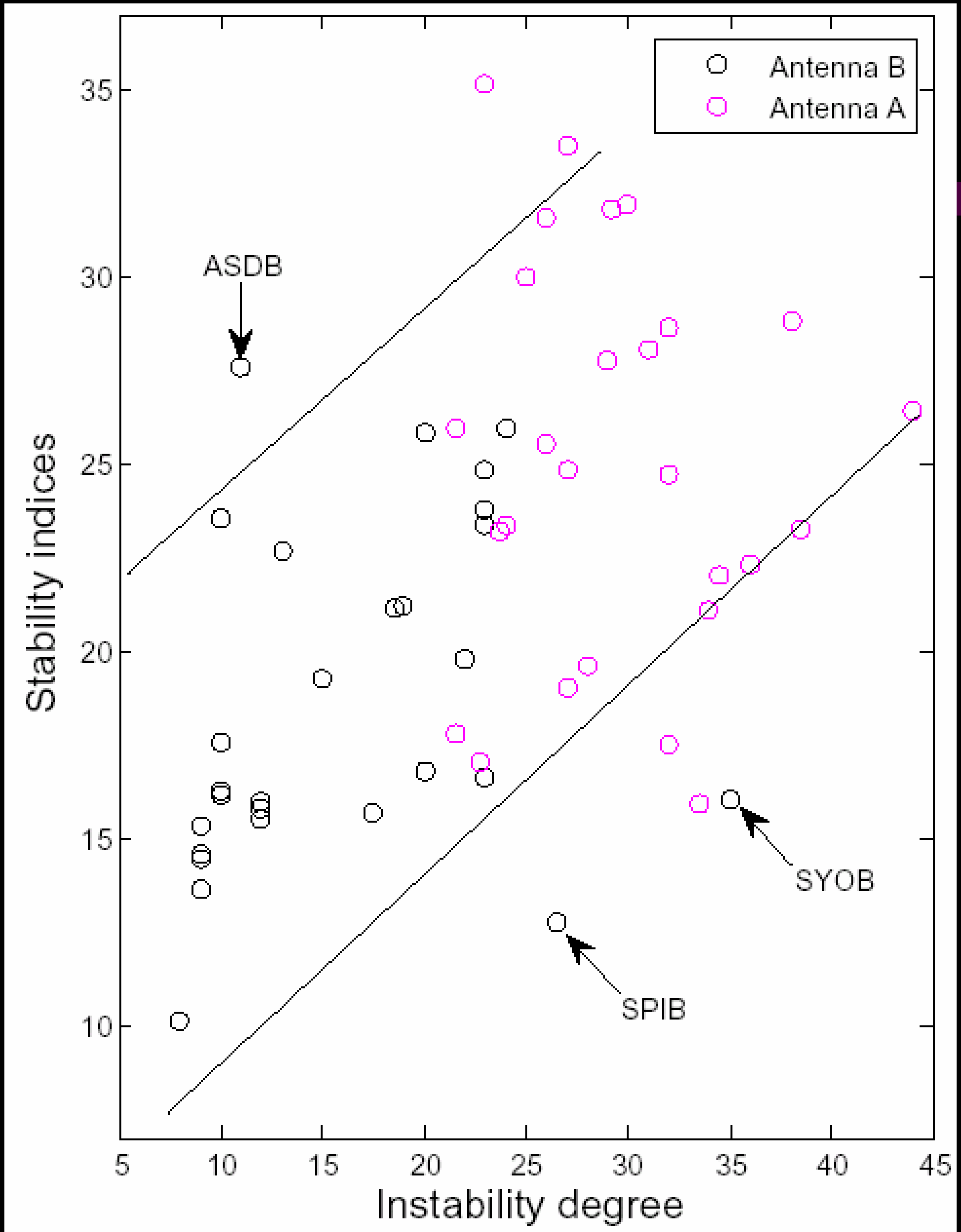
$$\frac{1}{100}\sum_{i=1}^{3}(ASd_i + Arate_i)\times Pc_i$$

Asd_i : Allan standard deviation (7 days) PCTi

$Arate_i$: Allan graph slope PCTi

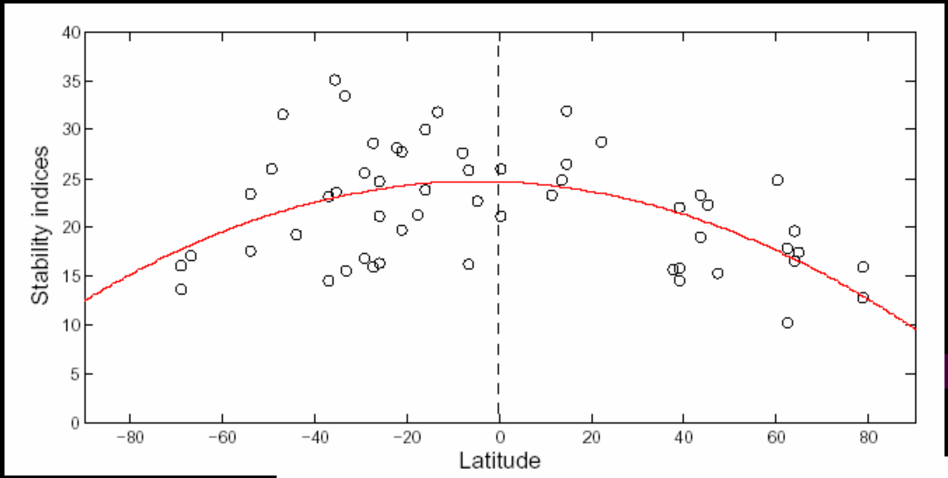
Pc_i : variance percentage PCTi

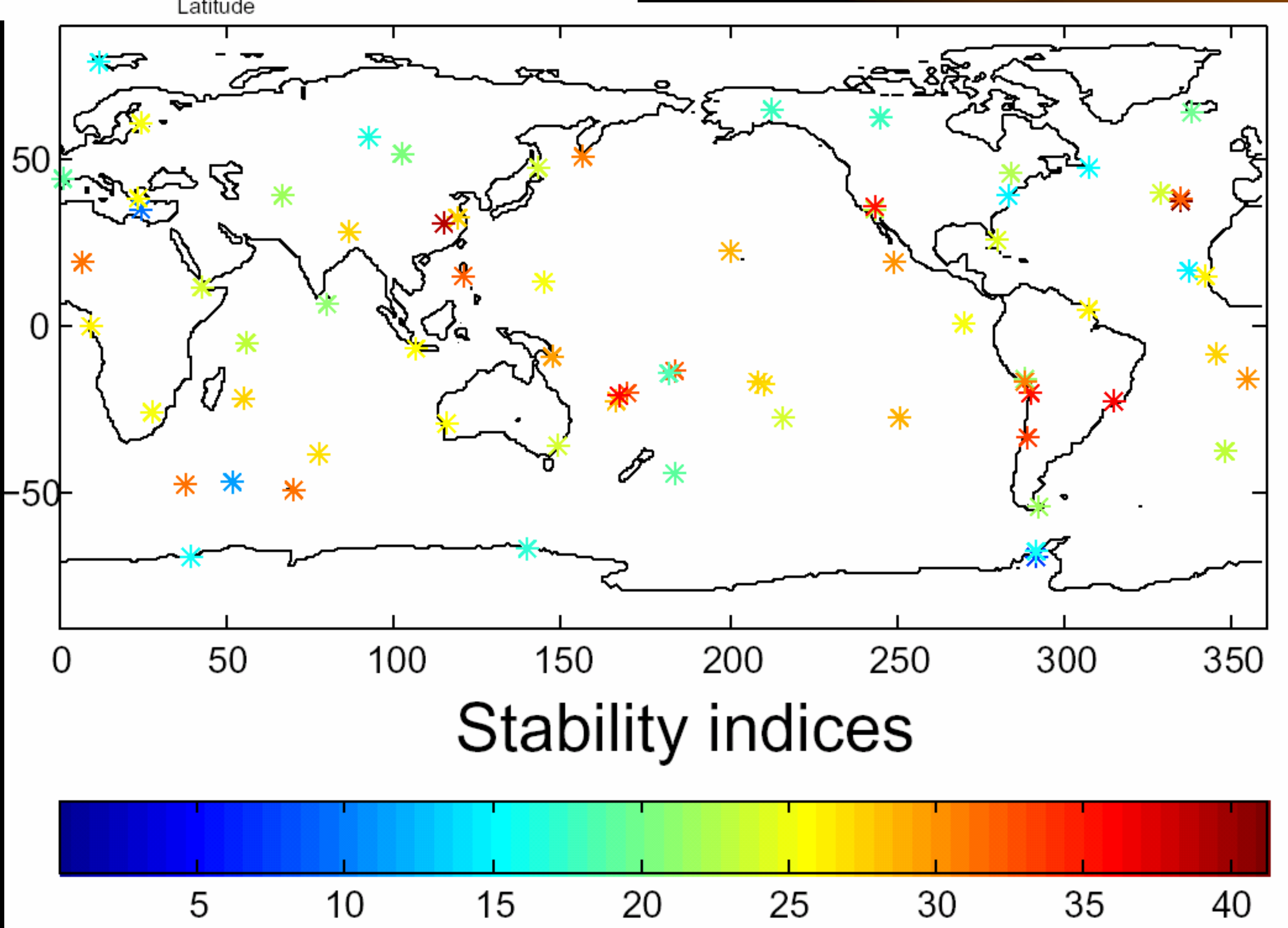
Level of noise, type of antenna and quality of the monument

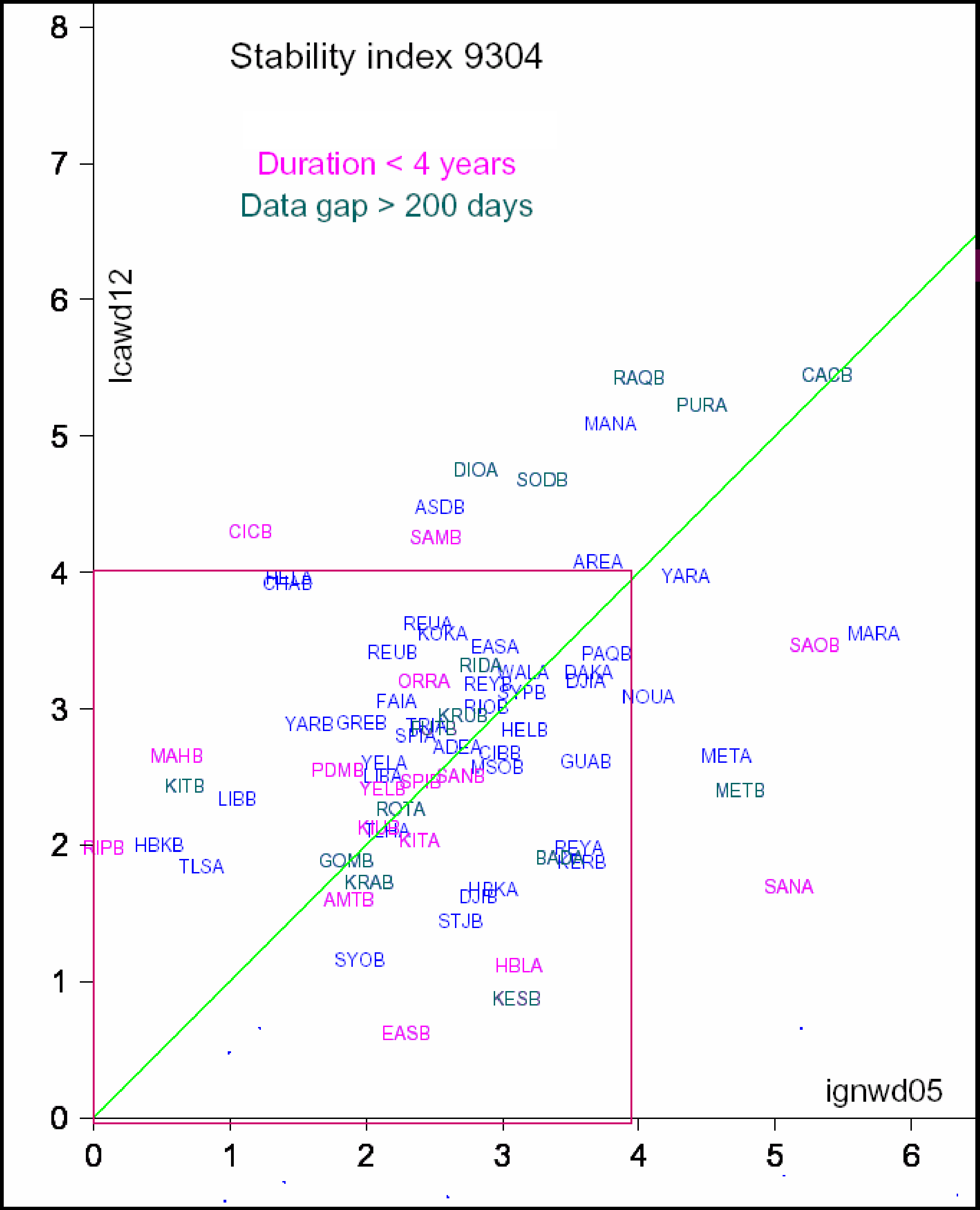


- Criteria from H.Fagard
- **A antenna** : bad monument index
- **B antenna** : except for 3 stations, it appears a correlation between the statistical level of noise and the quality of the monument

Latitude dependence of noise level







Stability indices for IGN and LCA solutions

- Disparity between the statistical qualities of the various solutions
- *Question:* do we have to take it into account for the combination?

Conclusions

- The IGN-JPL and INASAN solutions are fairly similar; The LCA solution is quite uncorrelated but generally show some similar annual terms;
- Results from the analysis of the non-linear non-seasonal part :
  - The DORIS dominant spectrum is white noise at the 10-45 mm level;
  - The 2001-2005 rejuvenation project is shown to improve the statistical stability;
  - The stability index reflects the statistical quality of a station.

⇨ Consideration of the derived weighting factor per station for the combination?

- The PCA applied in the time domain permits to identify independent noises.

Data analysis - scheme

$$\begin{pmatrix} dN \\ dE \\ dU \end{pmatrix}$$

Linear and annual and semi-annual terms removed

$$\begin{pmatrix} dN_c \\ dE_c \\ dU_c \end{pmatrix}$$

« noisy » interpolation

$$\begin{pmatrix} dN_{ci} \\ dE_{ci} \\ dU_{ci} \end{pmatrix}$$

PCAT

$$\begin{pmatrix} CP1, \%_{CP1} \\ CP2, \%_{CP2} \\ CP3, \%_{CP3} \end{pmatrix}$$

AV

$$\begin{pmatrix} AV_{CP1}^{7\,days}, l_{CP1} \\ AV_{CP2}^{7\,days}, l_{CP2} \\ AV_{CP3}^{7\,days}, l_{CP3} \end{pmatrix}$$

Informations on the type and level of noise of N,E,U components

Station stability index :

$$\frac{1}{100}\sum_{i=1}^{3}\left(AV_{CPi}^{7\,days} + l_{CPi}\right) \times \%_{CPi}$$