Doris models and solutions

Version 1.0

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1 Introduction

This document details the model equations for a complete solution using the Doris raw phase and pseudo-range measurements, available now at IDS in Rinex format [1]. The initial Doris phase processing used in precise orbit computations for Jason 2 at CNES was described in [2]. However, some improvements have been made, it is necessary to clarify some possible approximations made in the original solution. It is also necessary to allow the users to construct their own solution, using various approaches (for example, with directly the phase measurements, or using the phase variations). Also the current users Doris solutions need to be developed if the user wants to perform its own synchronization (pseudo-range processing).

The document first details the properties of the phase and pseudo-range Doris measurements, using the approach of [2]. Then some possibilities are explained for the solution of the measurement equations.

2 Measurements definitions and models

2.1 Single frequency measurements

Phase measurement definition, in meters :

$$\begin{aligned} Q &= \lambda\Phi_{re} \\ &= \lambda(\Phi_r - \Phi_e) + v_Q \quad \text{definition equation} \end{aligned} \tag{1}$$

Φ_{re} is the rinex phase measurement in cycles (L1 or L2 in the Rinex file). It is the difference between the receiver reference phase Φ_r and the phase of the received signal, which was Φ_e at the emission event.

$\lambda = c/f$ where f is the reference frequency for the considered frequency band (coefficient to convert the oscillator cycle count in receiver time). This is different from the 'true' frequency of the oscillator. For Doris, f has values 401.25 or 2036.25 MHz.

v_Q is the phase measurement error, the phase measurement noise is a few millimeters.

Phase measurement modelling, in meters :

$$\begin{aligned} Q &= c((\tau_r + h_r) - (\tau_e + h_e)) + Q_0 \\ &= c(t_r - t_e) + c(\delta_r^{rel} - \delta_e^{rel}) + c(h_r - h_e) + Q_0 \\ &= D_\Phi(t_r) + c(\delta_r^{rel} - \delta_e^{rel}) + c(h_r - h_e) + Q_0 \qquad \text{modelling equation} \end{aligned} \tag{2}$$

τ_r is the proper time for the receiver, τ_e is the proper time for the transmitter.

h_r is the receiver clock offset (usually it is modelled in Doris as a polynomial expression in τ_r). The receiver clock time for the reception event is $\tau_r + h_r$. The difference between $\tau_r + h_r$ and Φ_r/f is just a bias by definition of the receiver clock. This is also the case for the difference between $\tau_e + h_e$ and Φ_e/f.

t is the coordinate time for the reception ($_r$) or emission events ($_e$).

δ^{rel} is the difference between proper time and coordinate time for the receiver or the transmitter, $\tau = t + \delta^{rel}$. For the receiver (on board the satellite), it is a frequency offset with added periodic terms. For the transmitter (ground station), it is just a frequency offset. The corresponding expressions are shown in the appendix.

Q_0 is a common bias including the initial phase and a conventional time offset which may be present in the definition of the clocks relative to the USOs phase. Q_0 remains constant for a visibility pass when the receiver phase measurement is locked, it is different for each pass. In case of loss of lock during a pass, the value of Q_0 changes by an integer number of wavelentgh λ.

$D_\Phi(t_r)$ is the propagation time for the phase measurement between the transmitter and the receiver, expressed in meters, including atmospheric effects, phase centre and phase maps corrections, phase windup and Shapiro effect. It is a function of t_r (receiver coordinate time).

Pseudo-range measurement expression, in meters :

$$C = c((\tau_r + h_r) - (\tau_e + h_e)) + v_C \qquad \text{definition equation} \tag{3}$$

τ_r is the proper time for the receiver.

τ_e is here the emission time (proper time) corresponding to the pseudo-range, it is very close to the corresponding event for the phase.

v_C is the pseudo-range measurement error (rms values have a magnitude of several hundred meters).

$$C = D_C(t_r) + c(\delta_r^{rel} - \delta_e^{rel}) + c(h_r - h_e) \quad \text{modelling equation} \tag{4}$$

$D_C(t_r)$ is the propagation time between the transmitter and the receiver, expressed in meters, for the range measurement. It is a function of t_r (receiver coordinate time). The main difference with $D_\Phi(t_r)$ are the ionospheric contribution (opposite sign) and the phase windup effect which is not present for pseudo-range observables.

δ_e^{rel} can be supposed identical for phase and pseudo-range. Also h_e (which is a function of τ_e) is also supposed identical for phase and pseudo-range.

In the equations 4 and 2 there is a contribution of the ionospheric effect different for each frequency. These contributions are removed by the 'iono-free combination' of the measurements and corresponding models, see below.

2.2 Dual frequency case, iono-free combination

The Doris system uses two frequencies to remove the first order ionospheric effect, using a iono-free combination of the measurements (pseudo-range or phase).

For this dual frequency combination (iono-free combination), the model equations are similar. The values of τ_r and t_r are all identical for the two frequencies (the receiver processing is designed for synchroneous measurements). So the values of δ_r^{rel} and h_r are identical. For the ground transmitter, the corresponding emission events are not exactly synchroneous for the two frequencies, but the values of h_e and δ_e^{rel} can be considered identical for phase and pseudo range on both frequencies. For the geometry (iono-free D_C and D_Φ), if we suppose a 100 m differential effect due to iono, this produces a maximal error in the corresponding emission positions of 0.2 mm, which is negligible.

For the iono-free combinations, we have, with D_C or D_Φ corresponding to the iono-free propagations (no iono effect, and use of the iono-free reference phase centres, figure 1, and Q_0 including now all possible hardware biases (inter frequency biases) :

$$\begin{aligned} C &= D_C(t_r) + c(\delta_r^{rel} - \delta_e^{rel}) + c(h_r - h_e) && \text{pseudo-range} \\ Q &= D_\Phi(t_r) + c(\delta_r^{rel} - \delta_e^{rel}) + c(h_r - h_e) + Q_0 && \text{phase} \end{aligned} \tag{5}$$

The minimal measurement set to be used in the Rinex file is the receiver time $\tau_r + h_r$, and the corresponding iono-free combinations C obtained from C1 and C2, and Q obtained from λ_1L1 and λ_2L2.

This model is valid only for the beacons without K frequency factor. The case of shifted frequency beacons is detailed in the appendix.

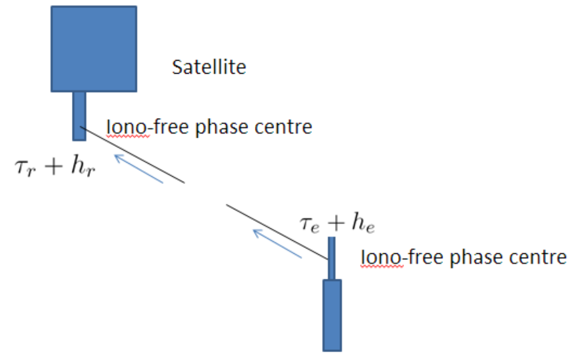

FIG. 1 – Signal propagation between ground station and satellite (iono-free combinations)

Other data like the clock offset present at the end of the Rinex epoch header (this value corresponds to $h_r + \delta_r^{rel}$, due to the synchronisation equations used for pseudo-range C, which do not have any relativity correction term), or the on board frequency, are obtained by the Diode navigator, or by the ground post processing. This implies that some systematic errors or unconsistencies may occur when using these data. However, these data are useful for simplified solutions, or for validation purposes.

3 Reception coordinate time t_r

For a given trajectory of the satellite, expressed in coordinate time, the objective is here to model correctly the phase measurement $D_\Phi(t_r)$, knowing the values of $\tau_r + h_r$, C and Q. So we have to estimate t_r, with a precision allowing a submillimer modelling (better than 10^{-7} m).

h_r can only be observed with the pseudo range measurements. Due to the important noise of the pseudo-range observations C, it is necessary to use a model for h_r, a snapshot solution is not realistic. An other reason is that for standard beacons the value h_e is unknown. h_e is only known for the time reference beacons (h_e=0 after correction with the bias and drift given in the Rinex file header), and these beacons are not in permanent visibility. For the time beacons, the relativity correction δ_e^{rel} must be set to 0.

The pseudo range equation 5 can be solved using a polynomial expression in τ_r for h_r, and a sufficient number of passes on reference beacons (typically more than two days are used, and a second degree polynomial). Due to the almost linear evolution between τ_r and t_r, the polynomial is usually expressed in t_r to simplify the coefficients identification.

It is important to notice that in the usual Doris solutions, the term δ_r^{rel} is not used in the pseudo-range equation. This term has a very important drift (frequency bias), and small periodic variations (a few centimeters). The periodic variations contributions are negligible for the reception time estimation. However the drift is not negligible and will be absorbed in this case by the adjusted polynomial. This means that in this case, for the phase processing, the relativistic correction δ_r^{rel} must be adapted to have no drift (like in GPS processing, where only the periodic terms are used in the GPS satellite clock correction).

4 Phase modelling Q

Now, knowing the on board clock offset h_r for all reception epochs t_r, it is possible to use the phase equation 5, where the only remaining measurement unknown parameters are h_e and Q_0 apart from other parameters like zenith troposphere delays or satellite orbit parameters which contribute to $D_\Phi(t_r)$ and will be adjusted in a global solution. To obtain information from this equation it is necessary to model h_e in a certain way, for example by adding constraints between successive epochs. In standard Doris Doppler processing, the beacon frequencies (or frequencies and drifts) are assumed to be constant during a pass, and are adjusted per pass. For the phase equation this corresponds to adjust a 1 or 2 degree polynomial function of τ_e (or t_e) to represent h_e. This was not clearly detailed in [2], the beacon polynomial is defined as a function of t_r, which leads to significative errors in station positioning when the beacon frequency bias is important.

For the relativistic corrections, the transmitter correction δ_e^{rel} is a bias and a drift (the beacon is fixed on ground), and so is not separable from the h_e polynomial expression. It can be corrected a priori, but this is not necessary.

The receiver correction is also mainly a bias and a drift, which cannot be separated from the polynomial expression of h_r as explained above. The periodic terms (due to eccentricity and J_2 contributions) can be modelled. In current Doris POD solutions, they are set to 0 (millimeter radial effect on the orbits), but are probably not negligible for station positioning.

Remark : in the current Doris 2.2 measurement file, these relativistic periodic terms are not taken into account, the on board frequency (derivative of h_r, or increments of successives values of h_r) is the only modelled term, as a low degree polynomial. This was not a problem at the beginning of Doris, but this approach has to be improved, as for satellites like Cryosat, the complete clock relativistic periodic bias amplitude contribution can reach 10 centimeters.

Of course, it is also possible to write directly the phase increments equations in Doppler mode (as it is the case for the standard Doris Doppler processing). The same properties hold for t_r, t_e, and for the related mean frequencies obtained with the h_e and h_r variations.

5 Choice between Doppler or phase processing

The Doppler processing corresponds to construct the difference of the phase equations from two consecutive epochs. This removes the common bias present in Q_0 and h_e. In this case, the variations of h_e and h_r correspond to the frequencies, and can be modelized as polynomials (apart from the relativistic effect for the receiver as explained above).

At the beginnig of the Doris system, the consistency between the receiver clock bias (used for estimation of t_r) and the on board frequency (derivative of h_r) was not imposed. In a first pass, t_r was identified for all measurements, using a synchronization signal similar to a pseudo range, and in a second pass, only the Doppler equations were processed, using an independent polynomial expression for the receiver frequency.

Now, the phase measurements are available, and the phase equation can be directly used. However, it is necessary to take into account the remainig modelling errors. The measurement have an intrinsic noise (a few millimeters for the phase), this noise is uncorrelated between successive epochs. Thus, from this point of view, the phase measurement equation direct processing is better (with diagonal weightings) : time differentiation for Doppler will produce correlations between successive measurements.

On the other hand the oscillator polynomial model is not perfect, and the oscillator has random errors strongly correlated with time (like a random walk). This error, which must be taken into account in the least squares weighting, is better handled using differences between successive phase measurements : for example this process minimizes the correlation between successive measurements in the case of a very low measurement noise, and a random walk for the oscillator. In the current system hardware, the main contributor to the errors is the oscillator, not the measurement noise. So it is theoretically better to use the phase variations than the phase (if we use only diagonal weightings).

However, if more sophisticated models are possible for the USO (for example stochastic models), the phase processing could be very interesting.

6 Possible solutions

We have seen that depending on the hypothesis, different solutions may be used to process the Rinex files measurements. The objective of this paragraph is to describe synthetically the possible solutions. Depending on the current software algorithms, some solutions are probably more suited for a simple implementation.

6.1 Decoupled solutions, USO polynomial models

A decoupled solution consists in solving for h_r in a first pass, and then for the remaining parameters (orbit, stations, ..) using the phase measurements, with known t_r values.

This is the current practice. Different possible configurations are shown table 6.1.

The polynomial $P_0(t_r)$ represents the h_r term (with or without including the relativity effects, the pseudo-range and phase modelling equations to be used must be consistent with this hypothesis).

rinex data/solutions	A	B	C
h_r^{rinex}	$h_r = h_r^{rinex}$	$h_r = P_0(t_r)$ adjusted on h_r^{rinex}	
C			$h_r = P_0(t_r)$ adjusted on C
Q (phase)	$P_1(t_r)$ adjusted	$P_0(t_r)$ fixed	$P_0(t_r)$ fixed
ΔQ (Doppler)	$\Delta P_1(t_r)$ adjusted	$\Delta P_0(t_r)$ fixed	$\Delta P_0(t_r)$ fixed

TAB. 1 – Different possible solutions

In table 6.1 the solution C in Doppler mode corresponds to the current solutions used at CNES to process the Rinex files. The older instruments are processed in a similar way, except that the model equations are directly constructed in Doppler mode.

Early Doris solutions were a mix between the solution A for the Doppler mode, but with a synchronisation corresponding to solution C : P_0 was not used for the Doppler processing, a new independent polynomial $\Delta P_1(t_r)$ was used.

The solution A needs a specific polynomial adjustment because the h_r^{rinex} is too noisy to construct correct phase measurements (the h_r value must be very smooth to achieve directly a phase measurement noise below 1 mm : the precision must be better than 10^{-12} seconds during a pass). In case of direct processing of the phase (polynomial $P_1(t_r)$), the constant term is undetermined together with the values of the pass biases Q_0 (equations 5).

For the synchronisation point of view, solutions A and B are close, the noise in h_r^{rinex} is sufficiently small to use directly this value for the estimation of t_r (however, the relativity hypotheses must be consistent for the phase processing in case of solution B because P_0 is used for the phase processing without any change). Solution A allows different models between the one which was estimated in the ground segment (for synchronisation use only), and the models for the phase processing, which give the final performance.

6.2 Coupled solutions, USO polynomial models

If we look at solutions B and C in table 6.1, we see that the estimation of the P_0 polynomial may be improved using simultaneously the pseudo-range and phase equations. This will not change significantly the synchronisation, but may improve the phase processing by allowing a better separation between ground and on board oscillators. This was not tested, one possible difficulty is that if the value of h_r is too important (this value may reach 5 s), the initial phase modelling could be very erroneous using a bad a priori for h_r, and may degrade the convergence of the process.

6.3 Other USO models

Usually the USOs (ground and on board) are modelled with polynomials. Better models (Markov processes) may be used, for example in solutions B or C, where this allows independent models for the USO mean term behaviour (during a pass, that is 10 minutes), and the long term behaviour represented by P_0 (typically 24 hours or more).

7 Conclusion and recommendations

This document shows the different possibilites which may be used for Rinex Doris data processing.

The main point is that it is preferred to use a Doppler formulation, by constructing the differences between successive phase measurements (in the case of USO polynomial modelling). An advantage of this approach is that the paramerization is identical to the current parameterizations using Doris 2.2 data. An other advantage is that the passes management is much easier.

Other formulations are possible, but need further investigations to achieve a correct performance.

For a simple solution, the receiver clock bias present in the Rinex file can be used for the synchronisation (construction of the correspondance between receiver time and coordinate time). However, one must be careful if this information is used also for the on board frequency estimation (consistency of the modelling hypotheses for the relativity frequency correction handling, and possible important noise). Using this synchronisation solution allows probably a correct Doppler solution using the phase variations, with minor changes in the existing software.

Références

[1] Rinex Doris 3.0, E. Lourme, CNES 2010, IDS website, ftp ://ftp.ids-doris.org/pub/ids/data/RINEX_DORIS.pdf

[2] Jason-2 DORIS phase measurement processing, F. Mercier, L. Cerri, J.P. Berthias Advances in Space Research, Volume 45, Issue 12, p. 1441-1454.

8 Appendix

8.1 Equations for shifted frequency beacons

The frequency shift is defined by an integer value K, given in the Rinex header [1]. This means that the nominal reference frequency of the beacon is shifted from the standard system frequency, and can be written as $f_K = (1 + a_K)f$ with f the nominal frequency.

The transmission time is constructed using the correct time evolution (that is, is exactly equivalent to the one obtained with a $K = 0$ beacon, driven by the same oscillator).

However, this is not the case for the phase measurement, which follows the same measurement equation expressed in cycles as the other beacons (equation 1). In order to be able to process these measurements in the same way as the standard case ($K = 0$), the corrected cycle count Q_{corr} (in meters) corresponding to the model equations 2 would be, with the shifted wavelength $\lambda_K = c/f_K$:

$$\begin{aligned} Q_{corr} &= \lambda\Phi_r - \lambda_K\Phi_e \\ &= \lambda_K(\Phi_r - \Phi_e) + (\lambda - \lambda_K)\Phi_r \\ &= \lambda_K(\Phi_r - \Phi_e) + c\frac{\Phi_r}{f}\frac{a_K}{1+a_K} \end{aligned} \tag{6}$$

The term $\frac{\Phi_r}{f}$ is proportional to the measured reception time $\tau_r + h_r$, which is directly the receiver measurement time present in the rinex file. Using this corrected Q_{corr} expression for the phase measurement, the shifted frequency beacons can be processed in the same way as all the other beacons.

8.2 Relativity effects

In this paragraph, we focus on the receiver relativity effect. The objective is to analyze the periodic terms. The complete effect for two events a and b can be formulated as :

$$\begin{aligned} \tau_b - \tau_a &= \int_{t_a}^{t_b} (1 + \frac{1}{c^2}(U - \frac{1}{2}v^2))dt \\ &= t_b - t_a + \int_{t_a}^{t_b} \frac{1}{c^2}(U - \frac{1}{2}v^2)dt \end{aligned} \tag{7}$$

U is the gravitational potential. For the Keplerian case, $U = -\frac{\mu}{||m(t)||}$, with $m(t)$ the position of the receiver at coordinate time t.

v is the velocity (inertial frame).

$U - \frac{1}{2}v^2$ is evaluated along the trajectory expressed in coordinate time t.

In equation 8.2, the integration along an orbit will produce a constant drift term (not perfectly constant with time for long durations, due to drag effects for example). This long term behaviour will be absorbed in the oscillator polynomial model.

There are different formulations to estimate the remaining periodic term. For GPS the expression is analytically developed for a Keplerian orbit, and can be expressed as $-2\frac{m(t).v(t)}{c^2}$ using the actual position and velocity of the satellite. However, this expression is not precise enough for LEO satellites.

The figures 2 and 3 show the results for the formulations. The formulations are : the standard GPS correction, estimation of equation with $U = -\frac{\mu}{||m(t)||}$, estimation of equation with U including the J_2 effect, and complete potential U. It is necessary to use the J_2 expression for U, and higher order terms have a negligible effect. The contribution is mainly at the orbital period and twice the orbital period.

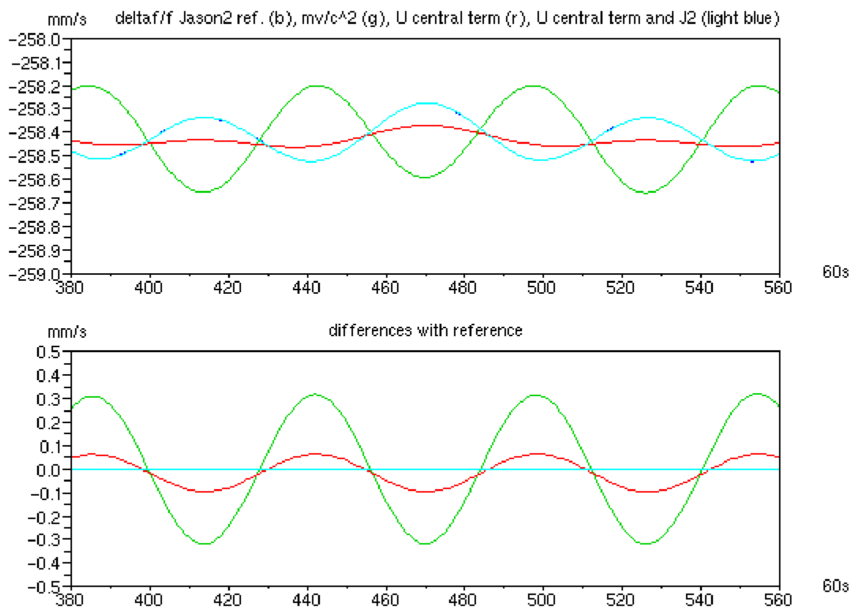

FIG. 2 – $\Delta f/f$ for Jason 2 , complete (blue), U central term (red), U central term and J_2 (ceil), GPS formula (green)

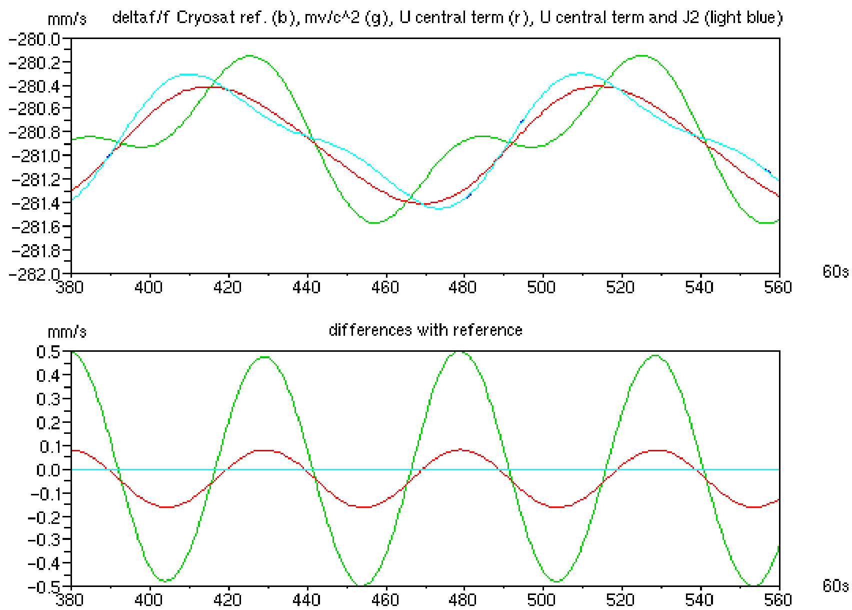

FIG. 3 – $\Delta f/f$ for Cryosat , complete (blue), U central term (red), U central term (ceil) and J_2, GPS formula (green)