# THE DOPPLER OBSERVATION EQUATION IN THE GINS SOFTWARE

**KEYWORDS: DOPPLER, DORIS, RINEX**

**SUMMARY: THIS DOCUMENT EXPLAINS HOW THE DORIS DOPPLER OBSERVATIONS ARE PROCESSED IN THE « GINS » SOFTWARE.**

| Le Rédacteur | Le Chef du Service de Géodésie Spatiale |
| --- | --- |
| **J.M. LEMOINE (JEAN-MICHEL.LEMOINE@CNES.FR)** | **R. BIANCALE** |

**Diffusion :**

IDS
CLS: L. Soudarin, H. Capdeville, S. Loyer
DCT/SI/GS R. Biancale, F. Mercier, J.M. Lemoine
DCT/SB/OR A. Couhert

# HISTORIQUE DES MODIFICATIONS

| Version | Date | Objet |
|---|---|---|
| 1 | 2015/03/25 | Création du document |
| 2.0 | 2015/04/01 | Correction d'une erreur dans la formule de la mesure de phase iono-free |
| | | |
| | | |
| | | |
| | | |

## REFERENCES

[1] Éric Gourgoulhon, *Relativité Générale*, http://luth.obspm.fr/~luthier/gourgoulhon/fr/master/relatM2.pdf

[2] Gérard Petit and Brian Luzum (eds.), *IERS conventions 2010*, http://www.iers.org/IERS/EN/Publications/TechnicalNotes/tn36.html

[3] *Modelling of DORIS 2GM instruments*, ftp://ftp.ids-doris.org/pub/ids/satellites/DORIS_instrument_modelling_2GM.pdf

# 1. BASIC EQUATIONS

## 1.1. CONVERSION BETWEEN PROPER TIME $\tau$ OF A CLOCK AND COORDINATE TIME $t$

**(1)** $$\boxed{d\tau \approx \left[1 - \frac{U}{c^2} - \frac{V^2}{2c^2}\right] dt}$$

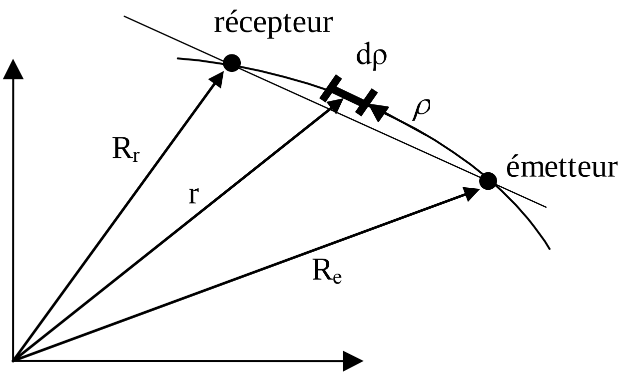
Where:

$U$ is the gravitational potential at the location of the clock

$V$ is the velocity of the clock in the coordinate reference frame

$c$ is the velocity of light in the void

## 1.2. TRANSIT TIME OF A PHOTON BETWEEN A POINT OF EMISSION AND A POINT OF RECEPTION




**(2)** $$\boxed{\Delta t_{transit} \approx \frac{\rho}{c} + 2\frac{\mu}{c^3} \ln\left(\frac{R_e + R_r + \rho}{R_e + R_r - \rho}\right)}$$

Where:

$\rho$ is the curvilinear trajectory of the photon; close at the first order to the geometrical distance between the emitter and the receiver

$\mu = GM$ , with $G$: gravitational constant, $M$: mass of the Earth

$R_e$ and $R_r$ are the geometrical distance of the emitter (resp. receiver) to the center of the reference frame, coincident with the center of mass of the Earth

In the following, the subscript $e$ will denote the emitter and $r$ the receiver.

Equations (1) and (2) are very simplified solutions (assuming in particular $\frac{V}{c} << 1$ and $\frac{U}{2c^2} << 1$) of the Schwarzschild equation:

**(3)** $$\boxed{ds^2 \approx \left(1 - \frac{2U(x,y,z)}{c^2}\right)c^2 dt^2 - \left(1 + \frac{2U(x,y,z)}{c^2}\right)(dx^2 + dy^2 + dz^2)}$$

which is itself a simplification of the Schwarzschild solution for a point outside of the Earth's masses, expressed in isotropic coordinates and in an inertial reference frame having its origin at the center of mass of the Earth (Ref. [1]):

$$ds^2 = \left(\frac{1 - \frac{U(x,y,z)}{2c^2}}{1 + \frac{U(x,y,z)}{2c^2}}\right)^2 c^2 dt^2 - \left(1 + \frac{U(x,y,z)}{2c^2}\right)^4 (dx^2 + dy^2 + dz^2)$$

## 2. THE DOPPLER OBSERVATION EQUATION

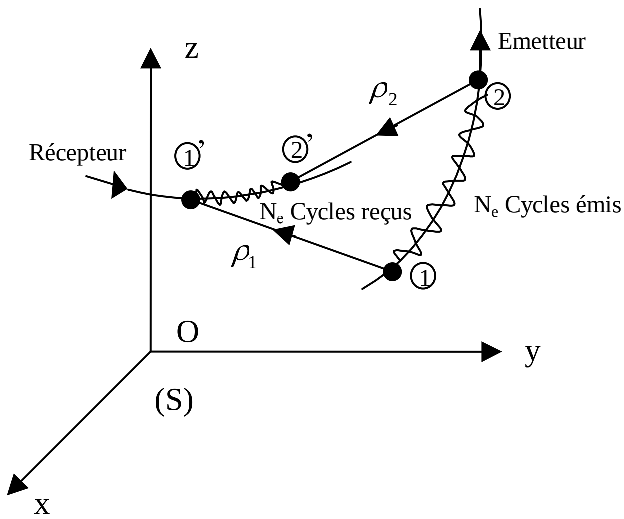
Let us define the 4 events:

① Emission of the 1st cycle by the emitter

①' Reception of the 1er cycle by the receiver

② Emission of the $N_e$ –th cycle by the emitter

②' Reception of the $N_e$ -th cycle by the receiver




The events are time-tagged $\tau_{e_1}$, $\tau_{e_2}$ in the emitter proper time scale, $\tau_{r_{1'}}, \tau_{r_{2'}}$ in the receiver proper time scale and $t_1, t_{1'}, t_2, t_{2'}$ in the coordinate time in the reference frame (S). Here the coordinate time that is considered is TCG (Geocentric Coordinate Time). TCG differs from the old TT scale and from TAI by a constant rate (1 + $L_G$), with $L_G = \frac{U_{GEO}}{c^2} = 6.969290134 \times 10^{-10}$ (cf. IERS conventions 2010, chapters 1 and 10, ref [2]).

During the proper time interval $\Delta\tau_r = \tau_{r_{2'}} - \tau_{r_{1'}}$, the receiver has received the $N_e$ cycles sent by the emitter, with $N_e = f_e \Delta\tau_e$, $f_e$ being the proper frequency of the emitter. The receiver is also equipped with an oscillator and during the proper time interval $\Delta\tau_r$, it has generated a number $N_r = f_r \Delta\tau_r$ of cycles, $f_r$ being the proper frequency of the receiver. The Doppler measurement is the count, by the receiver electronics, of the number of cycles of difference between $N_e$ and $N_r$:

$$\boxed{N_{DOP} = N_e - N_r} \quad \Rightarrow \quad \boxed{N_{DOP} = f_e \Delta\tau_e - f_r \Delta\tau_r}$$

In the RINEX files, this Doppler count is the difference between two phase measurements done at different time tags in the proper time of the receiver.

Let us express $\Delta\tau_e$ as a function of $\Delta\tau_r$, using coordinate time as an intermediary:

$$\Delta\tau_e = \tau_{e_2} - \tau_{e_1} \approx \left(1 - \frac{U_e}{c^2} - \frac{V_e^2}{2c^2}\right)(t_2 - t_1)$$ from (1), assuming $U_e$ et $V_e$ are constant over $(t_2 - t_1)$

We have :

$$\begin{aligned} t_2 - t_1 &= (t_2 - t_{2'}) + (t_{2'} - t_{1'}) + (t_{1'} - t_1) \\ &= -(t_{2'} - t_2) + (t_{2'} - t_{1'}) + (t_{1'} - t_1) \end{aligned}$$

$$t_{2'} - t_2 = \frac{\rho_2}{c} + \frac{2GM}{c^3} \ln\left(\frac{R_2 + R_{2'} + \rho_2}{R_2 + R_{2'} - \rho_2}\right)$$ from (2)

$$t_{2'} - t_{1'} = \Delta t_r \approx \left(1 + \frac{U_r}{c^2} + \frac{V_r^2}{2c^2}\right)\Delta\tau_r$$ from (1), assuming $U_r$ et $V_r$ are constant over $(\tau_2 - \tau_1)$; that is to say, in the case of an upward Doppler, for small orbit excentricities.

$$t_{1'} - t_1 = \frac{\rho_1}{c} + \frac{2GM}{c^3} \ln\left(\frac{R_1 + R_{1'} + \rho_1}{R_1 + R_{1'} - \rho_1}\right)$$ from (2)

Therefore:

$$N_{DOP} = f_e\left(1 - \frac{U_e}{c^2} - \frac{V_e^2}{2c^2}\right)\left[\left(1 + \frac{U_r}{c^2} + \frac{V_r^2}{2c^2}\right)\Delta\tau_r - \frac{\rho_2 - \rho_1}{c} + 2\frac{GM}{c^3}\left[\ln\left(\frac{R_1 + R_{1'} + \rho_1}{R_1 + R_{1'} - \rho_1}\right) - \ln\left(\frac{R_2 + R_{2'} + \rho_2}{R_2 + R_{2'} - \rho_2}\right)\right]\right] - f_r \Delta\tau_r$$

The above formula can be simplified in the terrestrial case, where $\frac{V}{c} << 1$ and $\frac{U}{2c^2} << 1$.

After these simplifications, which are not detailed here, we obtain the equations **(4)** that are used in GINS:

**(4)**

$$v_{measured} = \frac{c}{f_{e_N}}\left( f_{e_N} - f_{r_T} - \frac{N_{DOP}}{\Delta\tau_r} \right) + \Delta V_{IONO} + \Delta V_{TROPO} + \Delta V_{REL}$$

$$v_{theo} = \left(1 - \frac{U_e}{c^2} - \frac{V_e^2}{2c^2}\right)\frac{\rho_2 - \rho_1}{\Delta\tau_r} - \frac{c\left(\frac{N_{DOP}}{\Delta\tau_r} + f_{rT}\right)}{f_{e_N}}\frac{\Delta f_e}{f_{e_N}}$$

Where:

- the subscripts *e* denote the emitter and *r* the receiver;
- the subscripts *N* denote the nominal and *T* the true frequency;
- $v_{measured}$ is the measured relative velocity between the emitter and the receiver between the events 1') and 2'), based on the Doppler count $N_{DOP}$, corrected for the ionospheric, tropospheric and relativistic effects;
- $v_{theo}$ is the theoretical (or computed) velocity between the emitter and the receiver between the events 1') and 2'), corrected for a solved-for frequency bias per pass $\frac{\Delta f_e}{f_{e_N}}$ of the emitter;
- $f_{r_T} = f_{r_N} + \Delta f_r$ is an estimate of the true proper frequency of the receiver based either on a polynomial regression over the frequency offsets estimated during the passes over the master beacons (which have a quasi-nil proper frequency offset), or from the receiver frequency estimates found in the RINEX files. Beware: in that case the frequency estimation is not smooth enough, and a linear (or polynomial) interpolation has to be done between the first and last value of the RINEX file.
- $U_r$ and $U_e$ are the gravitational potential of resp. the receiver and the emitter
- $\Delta V_{REL} = \Delta V_{REL_H} + \Delta V_{REL_T}$ is the relativistic correction which is composed of two parts, the clock correction $\Delta V_{REL_H}$ and the transit correction $\Delta V_{REL_T}$:
  - $\Delta V_{REL_H} = \frac{1}{c}\left[U_r - U_e + \frac{V_r^2 - V_e^2}{2}\right]$
  - $\Delta V_{REL_T} = \frac{2\mu}{\Delta\tau_r\, c^2}\left[\ln\left(\frac{R_1 + R_{1'} + \rho_1}{R_1 + R_{1'} - \rho_1}\right) - \ln\left(\frac{R_2 + R_{2'} + \rho_2}{R_2 + R_{2'} - \rho_2}\right)\right]$

**IMPORTANT REMARKS:**

a) **Correction of aberration:** $\rho_\alpha$ is the geometrical distance between the emitter at time $t_\alpha$ and the receiver at time $t_{\alpha'}$. Since the measurements are made by the receiver, only $t_{\alpha'}$ is known. Therefore in order to compute accurately $t_\alpha$ and thus the position of the emitter at this time, a correction of aberration has to be done. It consists in computing an approximate value of the emitter-receiver distance $\rho^*_\alpha$ by evaluating the position of the emitter at time $t_{\alpha'}$, then determining $t_\alpha$ by applying the correction: $t_\alpha = t_{\alpha'} - \frac{\rho^*_\alpha}{c}$.
The process can be iterated but in general this is not necessary.

b) **Time scales**: The orbit computations are performed in a coordinate time, the TAI, which is equivalent, on the geoid, to the TCG ($TAI = (1-L_G)\ TCG$). The time interval that appears in equations (4) is an interval of onboard proper time. It is therefore very important to know in which time scale the measurements are provided. In the case of the RINEX DORIS, the measurements are supposed to be given in TAI. In fact, considering how the time-tagging is done by the DORIS project (see in particular ref. [3]), they are given in a kind of scaled proper time (let's call it $\tau_{DOR}$) . The proper onboard time $\tau_r$ is scaled by a low-degree polynomial in such a way that on the long run (i.e. for periods > 1 day) $\tau_{DOR}$ remains coherent with TAI. This absorbs the slowly varying receiver frequency offset $\Delta f_r$. But the relativistic fluctuations of $\tau_{DOR}$ with respect to TAI due to the orbit eccentricity are not taken into account. The rigorous conversion between receiver proper time and TAI is: $\Delta\tau_r = \left(1 - \frac{U_r}{c^2} - \frac{V_r^2}{2c^2} + L_G\right)\Delta t_{TAI}$

c) **Ionospheric correction**: In the RINEX files, the ionospheric correction has to be done by the users. A first-order iono-free measurement is built by combining the 400 MHz and 2 GHz measurements in the following way:

$$D_{iono-free} = \frac{\gamma D_{2GHz} - D_{400MHz}}{\gamma - 1}$$

Where $D_{2GHz}$ and $D_{400MHz}$ are the pseudo distance measurements on 2 GHz and 400 MHz, and $\gamma$ is the square of the frequency ratio: $\gamma = \left(\frac{f_{2GHz}}{f_{400MHz}}\right)^2$.

Converting to iono-free phase measurement on the 2 GHz channel using:

$$D_{iono-free} = \lambda_{2GHz} * L_{iono-free-2GHz},\ D_{2GHz} = \lambda_{2GHz} * L_{2GHz} \text{ and } D_{400MHz} = \lambda_{400MHz} * L_{400MHz}$$

One gets :

$$L_{iono-free-2GHz} = \frac{\gamma L_{2GHz} - \sqrt{\gamma} L_{400MHz}}{\gamma - 1} = L_{2GHz} + \frac{L_{2GHz} - \sqrt{\gamma} L_{400MHz}}{\gamma - 1}$$

Where:

- $L_{2GHz}$ is the phase measurement on 2 GHz
- $L_{400MHz}$ is the phase measurement on 400 MHz

When using these measurements, the 2 GHz phase centers can no longer be used as the end points of the measurement, the iono-free phase centers have to be used. The coordinates of the iono-free phase centers are given by the following formula:

$$\vec{r}_{2GHz,iono-free} = \frac{\vec{r}_{400MHz,2GHz}}{\gamma - 1}$$

Where:

- $\vec{r}_{2GHz,iono-free}$ is the vector from the 2 GHz phase center to the iono-free phase center
- $\vec{r}_{400MHz,2GHz}$ is the vector from the 400 MHz to the 2 GHz phase center

In the case of DORIS, $f_{2GHz} = 2.036250\text{GHz}$, $f_{400MHz} = 401.250\,\text{MHz}$, therefore $\gamma = 25.75325356$ and the iono-free phase centers are located a few mm away from the 2 GHz phase centers, in the direction opposite to the 400 MHz phase centers. The following table gives the phase center vectors for the different antennas (grey denotes the satellites and ground antennas for which there is no RINEX data; in that case the use of the iono-free phase center is not pertinent since a correction was introduced in the iono correction of the DORIS 2.1 and 2.2 measurements in order bring back the measurements to the 2 GHz phase centers):

| Unit = mm | Ref. point – 400 MHz | Ref. point – 2 GHz | 400 MHz – 2 GHz | 2 GHz – iono-free | Ref. point – iono-free |
|---|---|---|---|---|---|
| ALCATEL satellite antenna (SPOT-2) | 160 | 355 | 195 | 8 | 363 |
| STAREC satellite antenna (SPOT-3) | 152 | 315 | 163 | 6 | 321 |
| STAREC satellite antenna (SPOT-4) | 155 | 316 | 161 | 6 | 322 |
| STAREC satellite antenna (SPOT-5) | 153 | 315 | 162 | 6 | 321 |
| STAREC satellite antenna (TOPEX) | 161 | 317 | 156 | 6 | 323 |
| STAREC satellite antenna (ENVISAT) | 153 | 318 | 165 | 6 | 324 |
| STAREC satellite antenna (Jason-1) | 156 | 324 | 168 | 7 | 331 |
| STAREC satellite antenna (Jason-2) | 155 | 319 | 164 | 6 | 325 |
| STAREC satellite antenna (Cryosat-2) | 158 | 312 | 154 | 6 | 318 |
| STAREC satellite antenna (HY-2A) | 154 | 316 | 162 | 6 | 322 |
| STAREC satellite antenna (SARAL) | 156 | 314 | 158 | 6 | 320 |
| ALCATEL ground | 335 | 510 | 175 | 7 | 517 |

| | | | | | |
|---|---|---|---|---|---|
| antenna | | | | | |
| STAREC ground antenna | 0 | 487 | 487 | 19 | 506 |

d) **Emitter frequency bias**: In the computation of the effect of the emitter relative frequency offset, $\frac{\Delta f_e}{f_{e_N}} << 1$ has been assumed; therefore all terms in $\frac{\Delta f_e}{f_{e_N}^2}$ and $\frac{\Delta f_e^2}{f_{e_N}^2}$ have been neglected.

e) **Small terms**: In equations (4), the smallest terms are $\left(1 - \frac{U_e}{c^2} - \frac{V_e^2}{2c^2}\right)$ and $\Delta V_{REL_T}$. In the case of DORIS, they amount to maximum 11 and 6. $10^{-6}$ m/s respectively. Furthermore, since the emitters are located on ground, $\left(1 - \frac{U_e}{c^2} - \frac{V_e^2}{2c^2}\right)$ is constant per station. It is equivalent to a small frequency offset that will be absorbed by the adjustment of $\frac{\Delta f_e}{f_{e_N}}$. So equations (4) can still be simplified:

**(5)**
$$v_{measured} = \frac{c}{f_{e_N}}\left(f_{e_N} - f_{r_T} - \frac{N_{DOP}}{\Delta\tau_r}\right) + \Delta V_{IONO} + \Delta V_{TROPO} + \Delta V_{REL_H}$$

$$v_{theo} = \frac{\rho_2 - \rho_1}{\Delta\tau_r} - \frac{c\left(\frac{N_{DOP}}{\Delta\tau_r} + f_{rT}\right)}{f_{e_N}} \frac{\Delta f_e}{f_{e_N}}$$